On the Motion of a Uniformly Heated Drop in a Viscous Liquid under Gravity

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Abstract—The motion of a uniformly heated spherical drop under gravity is theoretically studied within the Stokes approximation. The Stokes and Hadamard–Rybchinsky formulas are generalized so that the temperature dependence of the viscosity can be found in a wide temperature range. Also, the drag force and the velocity of gravity fall are calculated for an arbitrary temperature difference between the surface of the drop and distant points. © 2002 MAIK “Nauka/Interperiodica”.

STATEMENT OF THE PROBLEM

We consider the motion of a uniformly heated hydrosol drop (particle) with a surface temperature $T_s$, in a viscous incompressible liquid under gravity. The liquid occupies the entire space, does not mix with the drop, and is at rest at infinity. A particle is considered to be heated (cooled) if its surface temperature differs from the temperature far away from it. Uniform heating can be associated with heat liberation during chemical reactions on its surface, the radioactive decay of the material, external effects, etc. If, for example, the particle is subjected to a monochromatic radiation of wavelength $\lambda_0$ and intensity $I_0$, it absorbs energy $\pi R^2 I_0 \kappa_n$ (where $R$ is the radius of the drop and $\kappa_n$ is the absorption factor [1, 2]), which is uniformly distributed over its volume. This statement is valid if the thermal conductivity of the drop is much higher than that of the environment and $\lambda_0 \gg R$. All the processes in the dropliquid medium system are quasi-stationary, because the thermal relaxation time of the system is small.

The heated surface of the drop influences the thermal physical characteristics of the surrounding liquid and, eventually, the velocity and pressure fields in its neighborhood.

Unlike [3–6], the author generalizes the Stokes and Hadamard–Rybchinsky formulas for the case of a uniformly heated spherical drop steadily moving in a viscous incompressible liquid. The temperature difference between the surface of the drop and distant sites, as well as the temperature dependence of the viscosity of the liquid, is assumed to be arbitrary.

Among all the parameters of liquid transport, the viscosity depends on temperature to the greatest extent, exponentially decreasing with growing temperature [7, 8]. The review of the available semi-empirical formulas and experimental data shows that the temperature dependence of the liquid viscosity $\mu$ is in a wide temperature range and with any desired accuracy can be described by the formula

$$\mu_e = \mu_\infty \left[1 + \sum_{n=1}^{\infty} F_n \left(\frac{T_e}{T_\infty} - 1\right)^n\right] \exp\left(-A\left(\frac{T_e}{T_\infty} - 1\right)\right), \quad (1)$$

where $A$ and $F_n$ are constants, $\mu_\infty = \mu_e(T_\infty)$, and $T_\infty$ is the liquid temperature far away from the particle (at $F_n = 0$, this formula reduces to the well-known Reynolds expression [7]). Hereafter, the subscripts $e$ and $i$ refer to the viscous liquid and heated particle, respectively; the subscript $\infty$ designates the parameters of the undisturbed flow at infinity; and the subscript $s$ refers to the parameters taken at the mean surface temperature $T_s$.

For water, $A = 5.779$, $F_1 = 2.318$, and $F_2 = 9.118$ with an accuracy of 2% or higher at temperatures between 273 and 363 K ($T_\infty = 273$ K).

It is assumed that the densities, thermal conductivities, and specific heat capacities of the liquid and the drop are constant. The drop moves slowly (small Reynolds and Peclet numbers) and retains the spherical shape. The latter statement is valid if the surface tension forces at the drop–environment interface far exceed the drag forces, which tend to distort the sphere. Analytically, the shape conservation condition is written as the inequality [9] $\sigma R \gg \mu_e \frac{U_e}{R}$, where $\sigma$ is the surface tension coefficient at the drop–environment interface and $U_e$ is the velocity of the particle. This inequality holds true for most liquids.

It is appropriate to relate the frame of reference to the center of the moving particle (the problem is reduced to the analysis of an infinite parallel flow with a velocity $U_\infty$ to be determined over the particle). The velocity and temperature distributions are symmetric about the Oz axis, which passes through the center of the particle and has the same direction as the incoming flow velocity. Therefore, we use the spherical coordinate system.
the stress tensor (normal and tangential) components must be complemented by the continuity conditions for
Laplacian; and (5)–(9) admit the separation of variables upon solving the hydrodynamic equations. The components of
the mass velocity and pressure were found in the form

\[ U_i(r, \Theta) = U_\infty G(r) \cos \Theta, \]

\[ U_\Theta(r, \Theta) = -U_\infty g(r) \sin \Theta, \]

where \( G(r), g(r), \) and \( h(r) \) are arbitrary functions depending on the radial coordinate \( r. \)

From the continuity equation, a relation between the functions \( G(r) \) and \( g(r) \) were determined. Finally, all the parameters and relations found were substituted into the appropriate Stokes equations. Eventually, we obtained the fourth-order ordinary differential equation for the function \( G(r) \) that is similar to that derived in [11]. Its solution was sought in the form of generalized power series. For the components of the mass velocity and pressure, we found

\[ U_i(r, \Theta) = U_\infty \cos \Theta[1 + A_1 G_1(y) + A_2 G_2(y)], \]  

\[ U_\Theta(r, \Theta) = -U_\infty \sin \Theta[1 + A_1 G_3(y) + A_2 G_4(y)], \]  

\[ P(r, \Theta) = P_\infty + \frac{\mu U_\infty}{R} \cos \Theta[A_1 G_5 + A_2 G_6], \]

\[ U_i(r, \Theta) = U_\infty \cos \Theta(A_1 + A_4 y^2), \]  

\[ U_\Theta(r, \Theta) = -U_\infty \sin \Theta(A_1 + 2 A_4 y^2), \]  

\[ P_i(r, \Theta) = P_0 + 10 \frac{\mu U_\infty}{R} A_4 \cos \Theta. \]