Diffractive Scattering in the Ericson Model for the $S$ Matrix

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Abstract—The elastic scattering of spinless charged particles on nuclei is considered within the strong-absorption model proposed by Ericson for the $S$ matrix in the angular-momentum representation. Our analytic method for summing partial-wave amplitudes, which is based on a generalization of the Abel–Plana formula, makes it possible to take into account the contributions from the possible singularities of the $S$ matrix in the right–hand half–plane of the complex–valued variable $t$. The uniform asymptotic behavior obtained in the present study for the scattering amplitude offers a fresh view on the origin of the diffraction patterns in the angular distributions of elastically scattered heavy particles. Special attention is given to Coulomb–nuclear interference (in particular, to refraction phenomena) in the case of scattering into the classically allowed region (illuminated region) and the classically forbidden region (shadow region). In contrast to other analytic results, our solutions to the diffraction problem within the Ericson model do not give grounds whatsoever to draw profound analogies either with Fresnel diffraction in optics or with the phenomenon of rainbow scattering in classical mechanics. © 2002 MAIK "Nauka/Interperiodica”.

1. INTRODUCTION

The existing descriptions of hadron–nucleus and nucleus–nucleus collisions at intermediate energies (for example, the scattering of heavy ions of energies ranging between a few MeV units to about ten MeV per nucleon) always take into account, in one way or another, nuclear absorption and refraction. These features of nuclear interaction may be reflected either in the properties of the real and imaginary parts of the optical potentials used or in models of the scattering matrix that satisfy some analytic requirements [1]. One of such models, that was formulated by Ericson [2] for the $S$ matrix that describes the elastic scattering of strongly absorbed particles, played an important role in the development of nuclear-diffraction theory.

Along with other studies that explored these realms [3–6], the applications of this theory (see, for example, [7]) formed a basis for obtaining qualitative insights into diffraction patterns found in the scattering of extremely light nuclei and heavy ions at collision energies above the Coulomb barrier. Above all, we mean applications to analyzing the effect of Coulomb–nuclear interference on the formation of the angular distributions of scattered particles and to drawing analogies between various modes of nuclear diffraction, on one hand, and the phenomena of Fresnel and Fraunhofer diffraction in optics and of rainbow scattering in classical mechanics, on the other hand. A comprehensive review of these issues can be found in [8] (see also references therein).

Owing to a simple dependence of partial elements of the model $S$ matrix in the angular-momentum representation on physical parameters, it is possible to obtain closed analytic expressions for the scattering amplitude (see, for example, [1]). In practice, one has to invoke various approximations, retaining only the “leading” terms in these expressions, where the interplay of basic physical ingredients is reflected only partly. It is obvious that such a partition into leading and nonleading contributions is a delicate point in solving the diffraction problem, which involves several characteristic quantities. These quantities specify the relation between the radius $R$ of nuclear-interaction (strong-absorption) region and the de Broglie wavelength $\lambda$ in the input scattering channel and the relation between $R$ and the thickness of the surface layer of this region, where there occurs a smooth transition from the transmission of incident partial waves without distortions to their complete absorption. In the case of charged–particle scattering, it is of paramount importance to single out correctly Coulomb repulsion effects, whose relative role is controlled by the Sommerfeld parameter.

In our opinion, available analytic results for diffractive scattering in the Ericson model are not free from drawbacks. Here, we mean, above all, not the
The objective of this article is to present new calculations of the elastic-scattering amplitude within this popular model and, then, to propose a possible interpretation of specific diffraction patterns in nuclear scattering (for example, owing to the refraction of incident waves). The analyses performed in [9, 10] for alpha-particle and pion scattering on nuclei in the energy region around 1 GeV furnished additional motivation to this study. The relevant expressions for the scattering amplitudes disregard some exponentially small contributions that cannot be ignored for not overly small values of the scattering angle. A detailed discussion of these contributions can be found in [11, 12].

Our further consideration will rely on previous theoretical elaborations [13] and on experience gained in deriving uniform asymptotic expansions [14] for typical diffraction integrals (Section 2). Section 3 is devoted to constructing an alternative description of oscillations of the Fresnel (or rainbow) type in the angular dependence of the ratio of the cross section for elastic scattering to the Rutherford cross section, $\sigma(\theta)/\sigma_R(\theta)$, for scattering angles in the illuminated region ($\theta < \theta_C$, where $\theta_C$ is the Coulomb angle corresponding to the motion of a charged particle in the Coulomb field of a force center along the grazing trajectory for which the distance of closest approach to this center is equal to $R$. In the same section, we formulate conditions under which there appears a dip in regular Fraunhofer-type oscillations of the elastic-scattering cross section $\sigma(\theta)$ for angles $\theta > \theta_C$ (shadow region). Within the method of complex angular momenta, this phenomenon was explained many years ago in [15].

2. CALCULATION OF THE AMPLITUDE FOR THE SCATTERING OF STRONGLY ABSORBED PARTICLES

2.1. Ericson Model and Related Quantities

In the model proposed in [2], the $S$-matrix elements appearing in the expansion

$$f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l + 1)(S_l \exp[2i\sigma_l] - 1) \exp[-(l + 1/2)\gamma] P_l(\cos \theta)$$

of the amplitude for the elastic scattering of spinless charged particles (here, $\sigma_l$ is the Coulomb phase shift and $k$ is the wave number) are approximated by the Fermi-like distribution

$$S_l \equiv \eta l \exp[2i\delta_l] = \left[ 1 + \exp \frac{l_0 - l}{\Delta} \right]^{-1},$$

where $l_0$ and $\Delta$ are model parameters. From this relation, we find the absorption coefficient

$$\eta_l \equiv |S_l| = \frac{1 - \eta_l(l)}{[1 + 4\Delta D_F(l) \sin^2(l_l/(2\Delta))]^{1/2}}$$

and the “nuclear” phase shift

$$\delta_l = \frac{1}{2} \arg S_l$$

for the corresponding absorptive shape function (compare with [16]).

Here, it is reasonable to introduce a nuclear (quantum) deflection function

$$\Theta_N(l) \equiv \frac{d\delta(l)}{dl} = \frac{1 - \eta_l(l)}{\Delta} \eta_l(l)$$

for the corresponding absorptive shape function (compare with [16]).

The quantities $S_l$ satisfy the unitarity condition

$$|S_l| \leq 1 \quad (l = 0, 1, 2, \ldots),$$

so that

$$\cos \frac{l_l}{\Delta} \geq 0.$$  \hspace{1cm} (8)

We recall that the grazing angular momentum $l_0$ is determined by the semiclassical relation

$$l_0 + 1/2 \equiv L = kR \sqrt{1 - B/E}$$

$$= kR \sqrt{1 - \frac{2n}{kR}},$$

where $B = Z_1 Z_2 e^2 / R$ is the height of the Coulomb barrier at the nuclear-interaction boundary, $E$ is the collision energy, and $k = \lambda^{-1}$ is the corresponding wave number. The Sommerfeld parameter $n = Z_1 Z_2 e^2 / \mu k$ (Coulomb parameter) depends on the charges $Z_1$ and $Z_2$ of colliding particles and on their reduced mass $\mu$.  

2) It is implied that, for the function $g_l$ specified at nonnegative integral values of $l$, an analytic continuation $g(l)$ to the complex plane of $l$ is implemented by substituting any complex values of $l$ in formulas of the type (2)–(4).