Phenomenological Analysis of the Experimental Data on the Photoproduction of $\eta$ Mesons off Nucleons

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Abstract—The initial stage of a phenomenological analysis of experimental data on the $\eta$ meson photoproduction off nucleons in the energy range from the threshold to 1.1 GeV is carried out based on a linear nonparametric model. The goal of this stage of the analysis is to obtain statistically reliable information about the partial waves that form the main characteristics of the process. The analysis uses the data of three laboratories about the angular distributions of $\eta$-mesons and their $\Sigma$ and $T$ asymmetries. The results of the analysis of the angular distributions demonstrate the presence of contradictions in the data obtained by different laboratories. The results of the analysis of the energy dependences of the polarization observables $\Sigma$ and $T$ show that the process regime probably changes in the vicinity of 0.9 GeV, which may be caused by the transition from the region of the $S_{11}(1535)$ and $D_{13}(1520)$ resonances to the region of the $D_{15}(1675)$ and $F_{15}(1630)$ resonances. © 2002 MAIK * Nauka/Interperiodica*.

1. INTRODUCTION

Investigation of the meson photoproduction processes off nucleons makes it possible to extract important information about the electromagnetic and strong decay properties of nucleon resonances from the characteristics of these processes. An increase in the interaction energy is accompanied by strong overlap of the resonances related to their large hadronic widths, which hinders reliable estimation of the resonance parameters. A unique possibility of avoiding these difficulties is offered by the study of the $\eta$-meson photoproduction process. This is related to the existence of a small number of resonances that decay into the $p\eta$ channel. We can believe that three resonances—$P_{11}(1440)$, $D_{13}(1520)$, and $S_{11}(1535)$—will play the dominating role immediately above the threshold of the $\gamma p \rightarrow \eta p$ reaction ($E_\gamma = 706.92$ MeV). The available data [1] indicate the possibility of determining, first of all, the characteristics of the $S_{11}(1535)$ and $D_{13}(1520)$ resonances.

The main goal of the analysis is to extract estimates of the photoproduction amplitudes, usually the multipole amplitudes, from the experimentally measured characteristics. This is justified by the fact that these amplitudes are directly related to the production of a nucleon resonance with certain quantum numbers corresponding to the given partial wave.

It is well known that the experimentally measured characteristics, namely, the differential cross sections $d\sigma/d\Omega$ of the process and the polarization properties (asymmetry $\Sigma$ for the linearly polarized beam of gamma radiation, asymmetry $T$ for the polarized proton target, polarization $P$ of the recoil nucleons), are bilinear forms of the complex amplitudes of the photoproduction process. Therefore, the analysis is naturally separated into two stages. The first stage is a linear regression problem. Solution of the problem gives information about the partial waves that contribute to the process. The problem is solved through expanding the observables into series, in this case, over the orthogonal Legendre polynomials, by the standard procedures [2] in order to determine the statistically significant terms of the expansion. In contrast to the expansion in power series with respect to $\cos \theta$, where $\theta$ is the meson emission angle in the center-of-mass frame (c.m.f.), the expansion with respect to the Legendre polynomials is related to their orthogonality. When the results of measurements for a sufficiently large number of emission angles are available, this property simplifies determination of the number of terms in the series providing the best description of data. The orthogonal polynomials give lower nondiagonal elements in the error matrix, which is important for interpretation of the diagonal elements as the errors of the regression coefficients. Thus, the first stage of the analysis determines the set of the linear regression coefficients that ensures the best (with respect to statistics) description of the experimental data. The data, obtained by various laboratories and used in the analysis, should be compatible—this fac-
tor is important for obtaining reliable estimates of the photoproduction amplitudes.\footnote{A similar analysis procedure was used in a series of papers aimed at the determination of the isotopic components of the multipole amplitudes for the $\gamma p \rightarrow \pi N$ process in the energy region of 300–400 MeV from experimental data on the pion photoproduction only (see, e.g., [3]).}

The second stage consists in solving a system of nonlinear (quadratic) equations with respect to real and imaginary parts of the multipole amplitudes. The number and the type of equations are determined by the results of the first stage. The right-hand sides of the equations with respect to the amplitudes represent the obtained estimates of the regression coefficients. Determination of the multipole amplitudes usually requires removing continuous and discrete ambiguities of the solutions caused by the individual specific features of the system of nonlinear equations, which, generally, make the system inconsistent.

2. THE ANALYSIS PROCEDURE

At the first stage of the phenomenological analysis of the experimental data, we used a nonparametric linear model where the observables are represented in terms of the expansion into series with respect to the Legendre polynomials $P_l(\cos \theta)$. For the differential cross section $d\sigma(\theta)/d\Omega$, the expansion assumes the form

$$\frac{k}{q} \frac{d\sigma(\theta)}{d\Omega} = \sum a_l P_l(\cos \theta),$$

where $k$ and $q$ are the momenta (in the c.m.f.) of the $\gamma$ quantum and $\eta$ meson, respectively. The regression coefficients $a_l$ are bilinear forms with respect to the real and imaginary parts of $E_{l\pm}$ and $M_{l\pm}$, which are, respectively, the electric and magnetic transition amplitudes to the final states with the total momenta $l \pm 1/2$. The amplitudes depend on the $\gamma$ quantum energy $E_{\gamma}$. The nonparametric model provides, generally, unbiased estimates of the regression coefficients, which ensures reliability of the further estimates of the multipole amplitudes of the process.

For the polarization observables, a similar expansion is applied to the statistics that include the observables. In general, the first stage of the analysis should provide statistically reliable information about the partial waves that form the main characteristics of the process. Reliability of the conclusions obtained at this stage determines reliability of the final estimates of the multipole amplitudes. We determined the number of the expansion terms providing the best description of the data on the basis of two statistical criteria. The expansion was restricted to the terms for which the coefficients at the Legendre polynomials significantly differ from zero. Here, we also checked that subsequent terms introduce no nonvanishing coefficients. Then, we determined the possibility of improving the quality of description by increasing the number of terms, as evaluated by the Fisher criterion [2]. Usually, both criteria led to the same conclusions.

The experimental data included the observables measured at a specified energy of the gamma quanta; that is, we performed a so-called energy-independent analysis. The energy-dependent analysis is based, as a rule, on the parametric models which do not ensure the obtaining of unbiased estimates. Of special importance is the question whether the experimental data measured by various laboratories form a unique general massive and therefore can be used to obtain final unbiased estimates of the multipole amplitudes of the process.

3. RESULTS OF THE ANALYSIS OF THE DIFFERENTIAL CROSS SECTION

First, we analyzed the results of measurements of the differential cross sections for the photoproduction of $\eta$ mesons off protons carried out at the Mainz accelerator [1]. These results include ten angular distributions measured at ten angles in the energy interval of the gamma-quanta from 715 to 790 MeV. Retaining only terms in the expansion with the coefficients at the Legendre polynomials significantly different from zero, we arrived at a three-term approximation that provided the best description of the complete set of experimental data on the differential cross sections. The Fisher criterion confirmed the correct choice of the three-term expansion. Table 1 presents the results of the three-term approximation. The errors of the individual regression coefficients are obtained from the diagonal elements of the error matrix with an allowance for the errors of the input data and the residual sum of squares that characterizes the quality of data description by the chosen model. The values of $\chi^2/n$, where $n$ is the number of degrees of freedom, characterize the statistical substantiation of the chosen linear model.

When analyzing the results in Table 1, attention should be paid to the following facts. First of all, the coefficient $a_0$ exceeds the coefficients $a_1$ and $a_2$ by one order of magnitude. Moreover, virtually all values of $a_1$ insignificantly differ from zero. This is clearly illustrated by the expansion in which the term with $a_1$ is omitted. It is clear from Table 2 that the absence of the term with $a_1$ does not spoil the description. The interpretation of these results requires consideration of the expressions for the regression coefficients in terms of the multipole amplitudes. When we use the