Micromaser Operation in Correlated Atoms

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Abstract—Stimulated emission of two-level atoms in a high-Q cavity is considered under conditions when pumping produces correlated many-atomic states. Based on the many-body problem, a kinetic equation is obtained for the Glauber–Sudarshan probability, which describes the field in the Fokker–Planck approximation. The statistics of light in this approximation are determined by the atomic correlation functions of the order of no higher than the second. The noise of light is found in the regime of micromaser operation for two types of pumping producing the initial separable states with a classical correlation and entangled states. It is shown that the presence of the initial diatomic correlation enhances the intensity noise. The entangled state of atoms is found from which nonclassical light is generated with a steady-state phase and noise, which can be almost completely suppressed in the low-frequency spectral region. © 2002 MAIK “Nauka/Interperiodica”.

1. INTRODUCTION

Micromaser operation of light [1, 2] has recently attracted a lot of attention. On the one hand, this is explained by the extensive study of the properties of micromasers and, on the other, by the possibility of the experimental verification of models in which the quantum description of the interaction of atoms with a field is required. The special feature of micromaser sources is the experimental realization of the conditions when the atomic relaxation is hardly manifested for the time of interaction of atoms with the field, so that the contribution from spontaneous processes is small, while stimulated emission dominates. The statistical properties of stimulated emission are determined by atomic correlations or atomic noise, for which an important role belongs to the pump mechanism. Upon regular pumping, a laser or a micromaser can generate light, which is characterized by a strict regularity of a photon flux in time or by sub-Poisson statistics of photons [3]. Such properties of light are of direct practical interest because in this case the noise is virtually absent.

However, the pump can produce correlated atoms. The theory of micromaser operation based on the standard Lamb–Scully approach [4] proves to be invalid for the description of a radiation source pumped in this way. This is explained by the fact that the Lamb–Scully method is based on a one-particle model, where the atoms are assumed to be independent, and their correlation, for example, of the “regular pump” type is introduced phenomenologically [3]. Our approach is based on the initial many-body problem describing the interaction of \( N \) two-level atoms with one mode of a high-Q cavity. In paper [5], where the many-body approach was also used, the nontrivial dynamics of operation of a micromaser field was found at the initial (but at the same time quasi-stationary) stage of the evolution for the Poisson statistics of injection of clusters consisting of excited independent atoms. Therefore, the question of how the initial state of atoms affects the properties of radiation is, in our opinion, of independent interest.

The process of obtaining the states with a multiparticle correlation is a separate problem, which we will not consider here. We restrict ourselves only to some types of the states that can be produced, for example, using protocols of the quantum theory of information. In this case, all \( N \) atoms are at once transferred to a correlated state, and therefore there is no need to discuss separately the question about the regularization inside such a cluster. Based on the many-body approach, we derive the kinetic Fokker–Planck equation for the field by using the formalism developed in paper [6]. However, because we assume that atoms do not interact with each other, although they are in the correlated state, no additional complications appear in Bardeen–Bogoliubov–Green–Kirkwood–Ivone chains. A special feature of the description of the field in the Fokker–Planck approximation is that the statistics of radiation are determined by atomic correlation functions of the order no higher than the second. It follows from this, in particular, that two different states with the same diatomic density matrix give the same statistics of radiation.

We consider the example of pumping that produces two types of initial states. The first type is a separable state with a classical correlation, which is described by the \( N \)-particle density matrix

\[
f(\ldots N) = \lambda_0 (|0\rangle \langle 0|)^{\otimes N} + \lambda_1 (|1\rangle \langle 1|)^{\otimes N},
\]

where...
where \( \lambda_1 + \lambda_1 = 1 \) and \( A^\otimes N \) is the tensor product \( A \otimes A \ldots \otimes A \). In this state, any pair of atoms has a two-particle density matrix of the form

\[
f(1, 2) = \lambda_0 |00\rangle\langle 00| + \lambda_1 |11\rangle\langle 11|,
\]

where \( \lambda_{0,1} \) are the level populations. It seems that it is impossible to prepare the \( N \)-particle state (1) within the framework of a simple pump model, when atoms are irradiated by a classical monochromatic wave. Indeed, consider a set of \( N \) independent atoms interacting with a classical wave. The initial entropy in this case is

\[
E_1 = -\lambda_1 \log \lambda_1 - (1 - \lambda_1) \log (1 - \lambda_1).
\]

The pumping process can be considered as a unitary evolution retaining entropy, which is \( S' = E_1 \) for the state described by expression (1). Therefore, no correlations will appear between the atoms. In other words, a classical monochromatic field does not produce correlations.

The states of the second type are inseparable or entangled, the particles having a special quantum correlation. Such states were obtained experimentally [7]. A nontrivial example of the decay of two atoms interacting with a common thermostat, when entangled atomic states appear even in the absence of a direct interatomic interaction, was considered in [8].

One of the states considered by us belongs to the GHZ (Greenberger–Horne–Zeilinger) class:

\[
|GHZ_N\rangle = \alpha |0\rangle^\otimes N + \beta |1\rangle^\otimes N,
\]

where \( |\alpha|^2 + |\beta|^2 = 1 \). For \( \alpha = \beta = 1/\sqrt{2} \), the degree of entanglement is maximum, while, for \( N = 2 \), the Einstein–Podolsky–Rosen (EPR) pair appears. The state (2) is pure, with a two-particle matrix (1). Assuming \( \alpha = 0 \), we obtain the case of \( N \) independent atoms, each of them being in the state \( |1\rangle \), which we consider as the upper state for definiteness. If the pump produces such atoms, then micromaser emission will have the sub-Poisson photon statistics, as in the case of a model with regular pumping, the Fokker–Planck equations being completely coincident in these two cases. This means that the monoatomic model of a maser with regular pumping is equivalent to the case considered by us, when all the independent atoms produced upon pumping occupy the upper level.

The second entangled state, which we consider here, has the form

\[
|\zeta\rangle = \alpha |bb\rangle + \beta |\Psi^+\rangle,
\]

where

\[
|\alpha|^2 + |\beta|^2 = 1, \quad |\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}},
\]

where \( b = 0, 1 \). Each atom in this state has coherence or polarization. Therefore, the phase of light generated in this case will be constant. We found the conditions under which the field generated from the cavity had the sub-Poisson photon statistics.

The main goal of our paper is to derive the kinetic equation for the operation field when pumping produces correlated atoms. Based on the equation obtained, we considered the characteristics of the steady-state micromaser operation.

The paper is organized as follows. In Section 2, we derive the kinetic equation in the Fokker–Planck approximation by the method of adiabatic exclusion of atomic variables. The behavior of atomic averages, which determine the field statistics, is considered in Section 3. In Section 4, we obtain the diffusion coefficients of the kinetic equation for different initial atomic states. In Sections 5 and 6, we calculated noise of light for two types of pumping producing classical and quantum correlation.

2. BASIC EQUATIONS

Consider \( N \) identical two-level atoms in a high-Q cavity, which interact with one cavity mode of the radiation field during the time \( T \). Let the frequency of a mode of the electromagnetic field be equal to the working transition frequency. For the micromaser operation, the approximations \( 1/T \gg \gamma \gg C \) are typical, where \( \gamma \) is the decay rate of atomic levels and \( C \) is the decay rate of the field mode. The density matrix \( \hat{F} \) describing atoms and the field at times shorter than \( T \), when the atomic relaxation can be neglected, satisfies the equation

\[
\frac{\partial}{\partial t} \hat{F} = [\hat{\vartheta}, \hat{F}],
\]

\[
\hat{\vartheta} = -i\hbar^{-1} V,
\]

\[
V = -i\hbar g (S_{10} b - S_{01} b^\dagger).
\]

Here,

\[
g = d \hbar^{-1} \sqrt{\frac{\hbar \omega}{2L^3 \epsilon_0}},
\]

\( d \) is the transition dipole moment; \( b^\dagger \) and \( b \) are the operators of photon creation and annihilation, respectively; and the atomic operators \( S_{xy} \) are defined by the relations

\[
S_{xy} = \sum_{a=1}^{N} s_{xy}(a),
\]

\[
s_{xy}(a) = |x\rangle \langle y|,
\]

\( x, y = 0, 1, \)