Theoretical Investigation of the \( p + \text{He} \rightarrow \text{H} + \text{He}^+ \) and \( p + \text{He} \rightarrow \text{H} + \text{He}^{++} + e \) Reactions at Very Small Scattering Angles of Hydrogen

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Abstract—A hypothesis is considered that the reactions \( p + \text{He} \rightarrow \text{H} + \text{He}^+ \) and \( p + \text{He} \rightarrow \text{H} + \text{He}^{++} + e \) at very small scattering angles of hydrogen can be used for the angular spectroscopy of electron correlations in a target. It is shown that this hypothesis is inconsistent. © 2002 MAIK “Nauka/Interperiodica”.

1. INTRODUCTION

A number of sufficiently fine experiments have been carried out over the last several years on the capture of an electron by a fast proton from a helium target, including those with a simultaneous transfer ionization process. With the use of a unique cold target recoil ion momentum spectroscopy (COLTRIMS), all independent kinematic characteristics of the final products of reactions were measured: the polar and azimuthal angles of hydrogen, as well as the momentum and energy of a \( \text{He}^{++} \) ion [1–3]. The azimuthal scattering angle \( \theta_p \) of hydrogen was as small as 0.1–0.5 mrad, which is about 100 times less than that obtained in earlier experiments of this type. The proton energy \( E_p \) ranged from 0.15 to 1.4 MeV.

The singly differential cross section \( d\sigma/d\theta_p \) of these reactions in the range of \( \theta_p \) from 10 to 1000 mrad represents a sufficiently smoothly and rapidly decreasing function, which is satisfactorily described within the continuum distorted wave formalism [4, 5]. At angles of \( \theta_p = 0.1–0.3 \) mrad, this curve attains its principal maximum (not counting relatively small Thomas peaks at appropriate scattering angles [6, 7] greater than 0.5 mrad).

The authors of the experimental works [1–3] suggested that the reaction \( p + \text{He} \rightarrow \text{H} + \text{He}^{++} + e \) in this range of extremely small scattering angles \( \theta_p \) can be used for obtaining new and extraordinary information on the structure of the wave function of the target in the momentum representation. The present paper is devoted to the theoretical analysis of this concept. Throughout this paper, we use atomic units.

2. THEORY

For brevity, we will call the reaction

\[ p + \text{He} \rightarrow \text{H} + \text{He}^+ \] (1)

a simple capture (SC) reaction and

\[ p + \text{He} \rightarrow \text{H} + \text{He}^{++} + e \] (2)

a transfer ionization (TI) reaction. We will also use the following notation: \( v_p(p_p) \) is the proton velocity (momentum), \( v_H(p_H) \) is the hydrogen velocity (momentum), \( k \) is the momentum of the escaped electron, \( K \) is the momentum of the residual ion, and \( E \) is the total energy of the system. In atomic units, the proton mass is \( m = 1836.15 \) and the ion mass is \( M = 4m \). In addition, we introduce the transferred momentum

\[ q = p_H - p_p = (m + 1)v_H - mv_p. \]

First, consider the TI reaction. In the notation introduced, the energy and momentum conservation laws in the laboratory system of coordinates are expressed as

\[ K + k + q = 0 \] (3)

and

\[ E = \frac{p_p^2}{2m} + E_0^{\text{He}} = \frac{p_H^2}{2(m + 1)} + \frac{k^2}{2} + \frac{K^2}{2M} + E_0^H, \] (4)

respectively. Here, \( E_0^{\text{He}} = -2.903 \) and \( E_0^H = -0.5 \). For convenience, we introduce the quantity \( Q = E_0^{\text{He}} - E_0^H = -2.403 \).

The proton energy \( E_p \) ranges from 0.15 to 1.4 MeV, which corresponds to \( v_p = 2.45–7.49 \). At the same time,
it follows from the experiments that the measured values of the ion momentum and the transferred momentum at very small angles \( \theta_p \) only amount to a few atomic units; this fact allows one to neglect the energies \( K^2/2M \) and \( q^2/2m \) as compared with other terms in Eq. (4). We stress that this can only be done at very small angles \( \theta_p = 0.1–0.5 \text{ mrad} \), when the helium ion remains at rest. At larger scattering angles, the proton–nucleon (pN) Coulomb interaction begins to play an increasing role, which significantly increases the transferred momentum and the momentum of the residual ion, which starts to move. Under the approximations made, it follows from (4) that

\[
\mathbf{v}_p \mathbf{q} = \frac{1}{2} \mathbf{v}_p^2 - \frac{1}{2} k^2 + Q.
\]

If we choose the proton velocity vector as the \( z \) axis, we have \( \mathbf{q} = (q_z, q_z) \), where

\[
q_z = \frac{\mathbf{v}_p \cdot k - 2 Q}{2 \mathbf{v}_p}.
\]

and \( q_z = m \mathbf{v}_p \sin \theta_p = m \mathbf{v}_p \Theta_p \). Note in passing that Eqs. (3)–(5) allow one to determine the total momentum of the electron provided that the momentum of the ions has been measured, to obtain appropriate constraints, etc.

For the SC reaction, Eqs. (3) and (4) are rewritten as

\[
\mathbf{K} + \mathbf{q} = 0
\]

and

\[
\frac{p_p^2}{2m} + E_0^\text{He} = \frac{p_0^2}{2(m+1)} + \frac{K^2}{2(M+1)} + E_0^\text{He} + E_0^\text{He}^*,
\]

where \( E_0^\text{He}^* = -2 \). Here, it is convenient to introduce a quantity \( Q = E_0^\text{He} - E_0^\text{He}^* = 0.403 \). Equation (6) is modified as follows:

\[
q_z = \frac{v_p}{2} + \frac{Q}{v_p}.
\]

Now, we pass on to the dynamics of the processes. Let us write the Hamiltonian of the system \( p + \text{He} \) as

\[
\hat{\mathcal{H}} = \hat{\mathcal{H}}_p + \hat{\mathcal{H}}_\text{He} + V_{p\text{He}},
\]

where

\[
\hat{\mathcal{H}}_p = \frac{p_p^2}{2m},
\]

\[
\hat{\mathcal{H}}_\text{He} = k_1^2/2 + k_2^2/2 + V_{N_e} + V_{N_e},
\]

\[
V_{p\text{He}} = V_{p_e} + V_{p_e} + V_{N_p},
\]

\[
V = V_{p\text{He}} + V_{N_e} + V_{N_e} + V_{ee}.
\]

Next, we use the following notation: \( \Phi_\mathbf{q} \) is the wave function of the helium atom at rest in the ground state; \( \Phi_\mathbf{p} \) is the wave function of the hydrogen atom in the ground state that moves at velocity \( \mathbf{v}_{N_e} \); and \( |\mathbf{K}, \varphi(\mathbf{k})\rangle \) is the wave function of the electron in the field of the ion \( \text{He}^+ \) with momentum \( \mathbf{K} \) (in the case of the SC reaction, one should take the function \( |\Phi_\mathbf{q}\rangle \) of the bound electron instead of \( |\varphi(\mathbf{k})\rangle \)).

We consider a helium atom in a singlet state; therefore, it is convenient to represent the amplitude of the TI reaction with regard to all the necessary symmetries as follows:

\[
\mathcal{F} = \langle \mathbf{p}, \Phi_0 | V_{p\text{He}}[1 + G(E)V_{\text{out}}]|\Psi_{\text{out}}\rangle,
\]

where

\[
G(E) = (E - \hat{\mathcal{H}}_p - \hat{\mathcal{H}}_\text{He} - V + i\varepsilon)^{-1}
\]

is the full Green’s function of the problem and \( |\Psi_{\text{out}}\rangle \) is determined from the equation

\[
[E - H_0 - (V - V_{\text{out}})]|\Psi_{\text{out}}\rangle = 0,
\]

where \( V_{\text{out}} = V_{ee} + V_{pN} \).

The amplitude (12) is exact, and the final state represents the wave function of two noninteracting electrons in the field of two centers that move relative to each other. Here, we consider the approximation of this complicated function by its asymptotic value, i.e., by the normalized combination of functions

\[
\langle \mathbf{r}_p, \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_N | \mathbf{p}, \Phi_0; \mathbf{K}, \varphi(\mathbf{k})\rangle,
\]

which is symmetric with respect to the electron coordinates. Even this simplification leaves open the problem of correct normalization, which requires the orthogonalization of all components. However, considering the velocity \( \mathbf{v}_p \) as a large parameter in the problem, one can neglect the cross-terms in the normalization integral and obtain an approximate normalization factor of \( N = 1/\sqrt{2} \).

Formula (12) clearly shows that the term in the first Born approximation in \( V_{p\text{He}} \) in problems with rearrangement is largely determined by the choice of the output potential \( V_{\text{out}} \), i.e., by the interaction in the final state; therefore, this term does not provide a suitable approximation for the amplitude \( \mathcal{F} \) even for a large energy \( E \). However, we will consider this term because it is this term that includes the simplest mechanism when one electron is captured immediately by a proton from an atom, while the other is emitted by the \( \text{He}^+ \) ion due to the shake-off process of the internal electric field in the atom. Indeed, leaving only \( V_{p\text{He}} \) in (12) and calculating the matrix element, we obtain

\[
\mathcal{F}_0 = -4\pi \sqrt{2} \int \frac{dx}{(2\pi)^2} \frac{\tilde{\Phi}_\mathbf{p}(\mathbf{x})}{|\mathbf{p} - \mathbf{q} - \mathbf{x}|} \left[ F(\mathbf{q}; 0; \mathbf{k}) + F(\mathbf{v}_p - \mathbf{q}; -\mathbf{v}_p + \mathbf{q} + \mathbf{x} - \mathbf{k}) - 2F(\mathbf{v}_p - \mathbf{x}; 0; \mathbf{k}) \right].
\]