Two-Dimensional Gas Motion in Nuclear-Pumped Laser Cells at Low Energy Deposits into the Gas

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Abstract—The distribution of the two-dimensional gas velocity inside ion-irradiated laser cells is considered for low ion energy deposits into the gas. It is shown that, if the energy deposit is smoothly nonuniform, the two-dimensional motion has two quasi-one-dimensional components: the longitudinal gas velocity is practically uniform across the cell and depends on the transversely averaged energy deposit, while the transverse velocity component depends on the difference between the local energy deposit and energy deposit averaged over the transverse direction. © 2003 MAIK “Nauka/Interperiodica”.

INTRODUCTION

The basic principle behind nuclear-pumped laser operation is the production of inversely populated laser levels in the gas medium via irradiation by ions that are products of nuclear reactions (usually, fission fragments from uranium layers; see review [1]). A nonuniform energy deposit from the ions into the gas leads to the redistribution of the gas density in the cell [2–7] and thereby adversely affects the lasing quality [2, 6]. One-dimensional calculations of the transverse gas motion in a gastight cell [2–6] and the numerical calculation of the two-dimensional gas density distribution over a flowing gas cell [7] have been carried out; however, the problem of two-dimensional distribution of the gas velocity has yet to be solved.

In this paper, the method of separation of variables is applied to analyze the two-dimensional gas motion inside a cell irradiated by fission fragments. The energy deposit from the fragments into the gas is assumed to be low (as compared with the internal gas energy). In this case, the gas velocity field is irrational and is described by the scalar potential satisfying the Poisson equation [8].

BASIC PHYSICAL PROCESSES

A nuclear-pumped laser is a gas-filled (gastight or flowing-gas) cell with thin uranium layers on its inner surface that irradiate the gas (Fig. 1). Here, cells with plane layers inside are considered in the two-dimensional approximation. For gastight cells [5], the x axis is directed along the optical one (the length of such cells is \( L \approx 1 \) m); for flowing-gas cells [6], the x axis is aligned with the gas flow and runs transversely to the optical axis (\( L \approx 0.1 \) m). The width of either cell is \( 2h \approx 0.01 \) m.

The key gasdynamic factor for such cells is the energy deposit from fusion fragments irradiating the gas from the uranium layers. The energy \( \delta Q \) of the fragments that is absorbed by a small gas volume \( \delta V \) for a time \( \delta t \) can be represented in the form [4, 5]

\[
\frac{\delta Q}{\delta V \delta t} = \frac{\Theta P_0 \rho(x, y, t)}{\gamma - 1} \psi(t) F(x, y, t),
\]

where \( \rho(x, y, t) \) is the gas density; \( \rho_0 \) is the initial gas density; \( P_0 \) is the initial gas pressure; \( \gamma = 5/3 \) is the ratio of the specific heat at constant pressure to that at constant volume; \( F(x, y, t) \) is the energy deposit function depending on the cell geometry, neutron flux distribution, and gas density (\( F(x, y, t) \approx 1 \)) [9]; \( \Theta \) is a dimensionless energy deposit parameter; and \( \psi(t) \) is the time-dependent neutron flux profile such that its integral taken over the irradiation time \( \tau \) is normalized to unity.

For gastight cells, \( \tau \) is the duration of the neutron pulse (\( \approx 1 \) ms); for flowing gas cells, \( \tau \) is the characteristic time of the gas flow: \( \tau = L/U_0 = 0.01 \) s, where \( U_0 \approx 10 \) m/s is the gas velocity at the cell inlet (\( x = 0 \)). The parameter \( \Theta \) is introduced as a thermodynamic measure of the energy deposit [4]; it equals the ratio of the fission fragment energy absorbed by a homogeneous ideal gas with a density \( \rho_0 \) in the cell for a time \( \tau \) to the internal energy of this gas. In practice, \( \Theta < 1 \) or \( \Theta = 1.0 \).
In this study, $\Theta$ is a small parameter: we assume that $\Theta \ll 1$.

The second small parameter is the Mach number $M$. In gas-tight cells, the transverse gas velocity is $w < h/\tau$ and the longitudinal velocity is $u < L/(2\tau)$; in flowing gas cells, $u = U_0$. For gases used in nuclear-pumped lasers (He or Ar at a pressure $P_0 = 1$ atm), the speed of sound $v_s \sim 10^4$ m/s; therefore, for gas-tight cells $w/v_s < 10^{-2}$ and $u/v_s < 0.5$, while for flowing gas cells $w/v_s < 10^{-2}$ and $u/v_s < 10^{-2}$. Thus, the gas pressure is practically uniform across the cell. Along flowing gas cells, the pressure is also uniform. However, for gas-tight cells, the longitudinal pressure can be considered to be uniform only if it is taken into account that $u \ll L/(2\tau)$ when $\Theta \ll 1$. The presence of considerable gas density differences at low Mach numbers, which is due to internal heat sources (see (1)), is a key feature of gas dynamics in nuclear-pumped lasers. Viscosity and heat conduction play a noticeable role only in the narrow near-wall layer. In most of the cell volume, their effect is negligibly small [2, 3, 6]. Under these conditions, the gas motion is virtually self-consistent thermal expansion over the cell volume; the self-consistency is conditioned by the pressure uniformity.

**GASDYNAMIC MODEL**

The gas is assumed to be ideal, inviscid, and non-heat-conducting, and the pressure is set equal to the cell-volume-averaged value:

$$p(x, y, t) = P(t) = \langle p(x, y, t) \rangle_v,$$

where $\langle \ldots \rangle_v$ means volume averaging.

The fission fragment energy $\Delta Q$ that is released in a gas volume element $V$ raises the internal gas energy $E = PV/(\gamma - 1)$ and does the expansion work $PAV$:

$$\Delta Q = \Gamma PV \Delta V + VAD.$$

In the limit $V \to 0$ and in view of the mass conservation condition ($\Delta VV \to -d\rho/d\rho$), we have

$$\Delta Q = \frac{dP}{d\rho} - \frac{\gamma P}{\rho} \frac{d\rho}{d\rho}.$$

subject to the continuity equation

$$\frac{1}{\rho} \frac{d\rho}{dt} + \nabla \cdot \mathbf{v} = 0, \quad \mathbf{v} = (u, w),$$

where $\mathbf{v}$ is the gas velocity.

The neglect of heat removal in gas-tight cells yields

$$\frac{dP}{dt} = \left(\gamma - 1\right) \frac{dQ}{\Delta V}.$$  

In flowing gas cells, $P = P_0$ under steady-state gas flow conditions.

The set of Eqs. (1)–(3) was solved for one-dimensional transversal gas motion in a gas-tight cell [3, 4]. In this case, the energy deposit function can be assumed to be fixed in Lagrangian coordinates: $F(x, y, t) = F_0(y_0)$, where $y_0$ is the Lagrangian coordinate defined by the equation $\rho dy_0 = \rho(y, t) dy$. In this case, the energy deposit is easily averaged and the gas pressure is determined at once. According to (1) and (3),

$$\frac{dP}{dt} = \Theta P_0 \psi(t) \int_0^h \frac{F_0(y_0)}{\rho_0} dy.$$  

because the energy deposit function $F(x, y, t)$ is normalized so that [4, 5] its volume-averaged value in a cell containing an unperturbed gas with a density $\rho_0$ is equal to unity: $\langle F(x, y, 0) \rangle_V = 1$. Hence,

$$P(t) = \Theta P_0 \int_0^t \psi(t') dt'. $$

At a specified gas pressure and energy deposit related to the Lagrangian coordinates, Eq. (2) is easily solved in the Lagrangian coordinates $(y_0, t)$ [3, 4]; the return to the Eulerian coordinates can be accomplished through the continuity equation. Thus, for one-dimensional gas motion, Eqs. (2) and (3), which express the first law of thermodynamics, allow one to completely solve the gasdynamic problem. At low Mach numbers $M$, the solution obtained is practically exact [3] and even at $M \to 1$ it well describes the dynamics of the smoothed density profile (on which, however, acoustic ripple is imposed [3]).

For two-dimensional flows, the situation is much more complicated. First, the energy deposit function is not fixed in the Lagrangian coordinates. Second, even if the gas density distribution is known, it is impossible, generally speaking, to describe the two-dimensional flow using the continuity equation alone: one must also invoke the Euler equation and consider the nonuniform pressure $p(x, y, t)$. Nevertheless, at low energy deposits, Eqs. (2) and (3) still make it possible to obtain an approximate two-dimensional distribution of the gas velocity. According to the Helmholtz theorem, a vector field is defined by its divergence and curl (up to a constant vector). The velocity field divergence is given by (2) and (3), while the velocity curl is given by the Friedrichmann equation [10], which in the two-dimensional case takes the form

$$\frac{d\omega}{dt} + \omega \nabla \cdot \mathbf{v} = \frac{1}{\rho^2} [\nabla \rho \times \nabla p], \quad \omega = |\text{curl}\mathbf{v}|.$$  

At low energy deposits ($\Theta \ll 1$), the differences in the gas velocities and densities (and, consequently, the pressure difference) are first-order perturbations in $\Theta$. 

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