1. INTRODUCTION

When an iron-ferrite film with perpendicular anisotropy is placed in an ac magnetic field (of frequency $\omega \sim 10^2$–$10^3$ Hz) normal to the film plane, the domain structure of the film becomes highly excited [1–3]. In this state, self-organization of moving domain walls occurs, bringing about the formation of various stable dynamic domain structures (DDSs), such as spiral domains (SDs) and systems of concentric ring domains. Each type of DDS is characterized by its region of existence, i.e., the ranges of magnetic-field amplitudes and frequencies over which the structure is stable. Furthermore, it has been found that the formation of a DDS depends on the parameters of the domain structure and material [4]. Domain structures superficially similar to DDSs (spiral and ring DDSs) but existing in a dc magnetic field were experimentally investigated in detail in [5, 6]. In particular, it was reported in [5] that SDs can also exist in the absence of an external magnetic field.

Up to now, DDSs have not been adequately studied theoretically. A dynamic system of ring domains was theoretically investigated in [7, 8]. The properties of SDs have been investigated theoretically only in the static state [9, 10]. In [11], spiral structures (vortices) were treated as defects in magnetically ordered media. Within a micromagnetic approach, it was shown that in the static state, the vortices in two-dimensional ferromagnets are due to exchange interaction. In this paper, we investigate the dynamic behavior of SDs in a film and the effect of an ac magnetic field and of various parameters of the film and of the domain structure on the SD stability.

2. MODEL OF THE SPIRAL DOMAIN

We will not discuss the initial process of the SD formation, because our aim is to study the stability of the SD. It is probable that the SD is formed from a stripe domain having one freely moving end. When moving, the end is subjected to gyrotropic force and the domain is curved into a spiral. In this paper, we assume that an SD with given initial geometric parameters (such as the outer radius and the pitch of the spiral) has already formed and investigate the time dependence of the outer radius of the spiral for various parameters of the film and external magnetic field.

The geometry of the problem is shown in Fig. 1. The magnetization in the domains is normal to the plane of the film and is reversed in a jump in going through a domain wall of zero thickness (in Fig. 1, the white and grey domains have opposite magnetization directions). This approximation is valid for films of a high quality factor $Q = K/2\pi M^2$, where $K$ is the uniaxial magnetic anisotropy constant and $M$ is the saturation magnetization. Therefore, the magnetostatic energy of space charges is ignored and the uniaxial-anisotropy and exchange-interaction energies are included effectively in the surface energy density $\gamma$ of the domain walls. Experimentally, SDs were observed to have the shape of an Archimedean spiral; deviations from this shape took place only at the periphery of an SD and were pro-
duced, perhaps, by the SD surroundings. In calculations, we assumed the spiral domain walls to be Archimedean spirals of pitch $P$.

Within these approximations, the SD energy [in units of $(2\pi M)^2 h^2$] in an infinite film was found in [9] to be

$$E = E_H(m, r) + E_W(r, \beta) + E_M^I(m, r) + E_M^s(m, r, \beta),$$

where

$$E_H(m, r) = H(1 - m)r^2$$

is the Zeeman energy,

$$E_W(r, \beta) = \frac{1}{\pi} r^2 \beta$$

is the domain-wall energy,

$$E_M^I(m, r) = -\frac{(1 - m^2)}{r} r^2 + \frac{(1 - m^2)}{4\pi} r^2$$

is the long-range part of the magnetostatic energy, and

$$E_M^s(m, r, \beta) = \frac{4r^2}{\pi^2 \beta} \sum_{n=1}^{\infty} \sin^2 \left(\frac{\pi n}{2}(1 + m)\right)$$

$$\times \left[1 - \left(1 - K(r)e^{-n\beta} - L(r, n\beta)\right)\right]$$

is the short-range part of the magnetostatic energy. Here, we introduced the following notation:

$$K(r) = \frac{1}{8r^2} \left(1 - 2 \ln \left[4r\right] \right) + \frac{1}{128r^4},$$

$$L(r, n\beta) = \frac{1}{\pi r} E_1(n\beta) + \frac{1}{8r^2} E_3(n\beta) + \frac{1}{8\pi r} E_3(n\beta),$$

$$E_1(x) = \frac{(k - 1)!}{x^k},$$

where $h$ is the film thickness, $r = R/h$ is the reduced outer radius of the SD, $H = H_0/4\pi M^2$ is the reduced amplitude of the external magnetic field, $m = \bar{M}/M$ is the reduced average magnetization of the spiral, $\sigma$ is the surface energy density of domain walls, $l = \sigma/4\pi M^2 h$ is a characteristic length of the material (normalized to the film thickness), $p = P/h$ is the reduced pitch of the spiral, and $\beta = 2\pi h/P$ is the inverse pitch (wave vector) of the spiral.

Experiment shows [1–3] that application of an ac magnetic field $H = H_0 \sin \omega t$ primarily affects the outer radius of the dynamic SD. We derived an equation describing the dynamics of this geometric parameter under the assumption that the SD is a stripe domain (of uniform width) curved into a spiral. In this case, the outer radius of the SD is determined only by the length $L$ of the stripe domain. The SD length is varied as the outer end of the stripe domain moves along a trajectory described by the equation of an Archimedean spiral. This process is accompanied by a change in the SD energy due to energy dissipation in the domain walls of the outer end of the stripe domain:

$$\frac{dE}{dt} = \frac{\partial E}{\partial r} \frac{dr}{dt} = -k_v^2,$$

where $\nu = d/Ldt = (\partial L/\partial r)dr/dt$ is the velocity of the outer end of the stripe domain, $k = 2\pi ph/\mu$ is the coefficient of viscosity, and $\mu$ is the mobility of domain walls.

The final equation of motion of the outer radius of the SD was found to be

$$k^* \frac{dr}{dt} = -\left(\frac{\partial E_W(r, \beta)}{\partial r} + \frac{\partial E_M^I(m, r, H \cos(\omega t))}{\partial r} + \frac{\partial E_M^s(m, r, \beta)}{\partial r}\right),$$

where $k^* = k(1 + \beta^2 r^2(t))$ is the effective coefficient of viscosity.

We solved Eq. (9) numerically using the fourth-order Runge–Kutta method for a given initial SD radius. The following parameters were varied: the reduced pitch $p$ of the spiral, the characteristic length $l$, and the reduced amplitude $H$ and frequency $\omega$ of the external field. Our main concern was to investigate the effect of these parameters on the behavior of the SD and its geometric parameters.

3. RESULTS AND DISCUSSION

First, we performed calculations for the static case ($H = \text{const}$); the results obtained are identical to those presented in [9]. In particular, we found the field $H^{(\text{col})}$ at which the SD collapses and the field $H^{(\infty)}$ ($H^{(\text{col})} > H^{(\infty)} > 0$) at which the SD radius tends to infinity (here and henceforth, the SD radius is taken to mean its maximum outer radius). At certain values of the pitch of the spiral and the characteristic length of the material of the film, the SD radius was found to be finite at $H = 0$. In [10], it was shown theoretically that the SD can exist in the absence of an external magnetic field, but with the energy minimum being less pronounced. Figure 2a shows the dependence of the SD energy on its outer radius for $p = 1.9$ and $l = 0.2$ in a zero external field. It can be seen that the SD energy is minimum at the SD radius $r_0 = 55$. A small change in the pitch of the spiral...