Transition to Fully Developed Chaos in a System of Two Unidirectionally Coupled Backward-Wave Oscillators

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Abstract—The chaotic dynamics of a system of two unidirectionally coupled backward-wave oscillators (BWOs) is studied in the case when a signal from the driving BWO in (periodic or chaotic) self-modulation mode is applied to the driven oscillator, which exhibits strong periodic self-modulation in the autonomous case. The oscillation evolution with the amount of coupling is traced. The use of a chain of coupled BWOs is shown to significantly reduce the threshold of transition to the regime of wide-band chaotic oscillations with a uniform continuous spectrum (so-called fully developed chaos), which is of interest for applications. © 2003 MAIK “Nauka/Interperiodica”.

High-power sources of microwave chaotic oscillations are promising for a wide range of applications, such as radar detection, plasma heating in systems of controlled thermonuclear fusion, advanced communications based on dynamic chaos, etc. Backward-wave oscillators are among the most extensively studied complex-dynamics vacuum electronic devices: their capability to generate chaotic oscillations was predicted and experimentally confirmed as early as in the late 1970s [1, 2]. Recent studies of BWO operation [3–7] showed that, as the beam current increases, regular and chaotic oscillation regimes alternate in a complex manner until the system comes to exhibit highly irregular wide-band chaotic oscillations with a fairly uniform continuous spectrum. This mode has received the name fully developed chaos. Evidently, it is the most favorable regime for the applications mentioned above. However, fully developed chaos arises when the electron beam current is considerably in excess (by more than 30 times) of the starting value, which is hard to provide in reality. Experiments with nonrelativistic [2, 8] and relativistic [9–11] BWOs are usually carried out on specially constructed setups with an extended phase length. In this case, additional difficulties associated, for example, with beam focusing may emerge.

In this study, we will show that two coupled oscillators may reduce the developed chaos threshold. The nonlinear dynamics of such a system is numerically studied by using the well-known equations from the nonstationary nonlinear theory of BWO (see, e.g., [1, 3–7]):

$$\frac{\partial^2 \theta_{1,2}}{\partial \xi^2} = -L_{1,2}^2 \text{Re}[F_{1,2} \exp(i\theta_{1,2})],$$  \hspace{1cm} (1)

$$\frac{\partial F_{1,2}}{\partial \tau} - \frac{\partial F_{1,2}}{\partial \xi^2} = \frac{L_{1,2}^2}{\pi} \int_0^{2\pi} \exp(-i\theta_{1,2}) d\theta_0.$$  \hspace{1cm} (2)

Equations (1) describe the motion of electrons in the field of an electromagnetic wave, while Eqs. (2) represent the nonstationary excitation of the slow-wave structure by a slowly varying current. The subscripts indicate the serial number of an oscillator in the chain. In Eqs. (1) and (2), \(\theta_{1,2}\) are the electron phases with respect to the wave; \(\theta_0\) are the initial phases; \(F_{1,2}\) are the slowly varying dimensionless amplitudes; and \(\xi\) and \(\tau\) are the dimensionless coordinate and time, respectively. Note that the equations of motion are written under the assumption of a small variation of electron energy during the interaction (see, e.g., [1, 6]) and apply to both the relativistic and nonrelativistic cases. The dynamics of a partial oscillator is governed by the sole bifurcation parameter \(L = 2\pi CN\), where \(C\) is the Pierce gain parameter and \(N\) is the phase length of the system. If one oscillator is taken to drive the other, the boundary conditions for Eqs. (1) and (2) can be set as

$$\theta_{1,2} \bigg|_{\xi=0} = \theta_0 \in [0; 2\pi], \quad \frac{\partial \theta_{1,2}}{\partial \xi} \bigg|_{\xi=0} = 0,$$  \hspace{1cm} (3)

$$F_1(\xi = 1) = 0, \quad F_2(\xi = 1) = RF_1(\xi = 0),$$

where \(R\) is the coupling parameter, which may be considered real without loss of generality.

Let us recall the basic results of the investigation into the complex dynamics of an autonomous oscillator [3, 4]. The self-excitation of oscillations starts at \(L \approx 1.98\). Stationary single-frequency oscillation modes exist within the range \(1.98 < L < 2.9\). At \(L = 2.9\), the stationary regime becomes unstable and gives way to periodic self-modulation. As the parameter \(L\) increases to
that are synchronized by an external harmonic action, effects typical of finite-dimensional chaotic systems; however, it is safe to predict the presence of all the nal signal. This phenomena calls for further investigation. As chaos was found to be suppressed with growing external signal on a chaotic BWO was studied in [8], where single-frequency mode, we face the well-known problem of synchronization by an external harmonic signal. Note that, if the driving oscillator operates in the steady mode of either oscillator. For example, one may consider the action of a periodic-mode oscillator on an oscillator operating in the chaotic mode and vice versa, interaction between two chaotic-signal sources, etc. Note that, if the driving oscillator operates in the steady single-frequency mode, we face the well-known problem of synchronization by an external harmonic signal. The problem of synchronization of a periodic oscillator has been studied in most detail; in general, this problem is qualitatively similar to the classical problem of the vibration theory when a harmonic force acts on a self-oscillating system [13, 14]. The effect of a harmonic signal on a chaotic BWO was studied in [8], where chaos was found to be suppressed with growing external signal. This phenomena calls for further investigation; however, it is safe to predict the presence of all the effects typical of finite-dimensional chaotic systems that are synchronized by an external harmonic action, such as the locking or suppression of the fundamental frequency of chaotic oscillations, etc. [15].

Since the primary goal of this study is to see whether the developed chaos threshold may be reduced, we will first consider the situation where both oscillators operate in the periodic self-modulation mode. The evolution of the oscillatory regimes with the amount of coupling seems to be the following. As $R$ grows, oscillations become chaotic basically through the decay of the quasiperiodic motion. Figure 1 presents typical phase portraits and output signal spectra for (a) the first and (b–f) second oscillators. The two-dimensional projection of the phase portrait is constructed for $F_{out} = [F_1, x(\xi = 0)]$ by the method of delays (Packard–Takens method) [15]. The values of the bifurcation parameters $L_1 = 4.0$ and $L_2 = 4.5$ are chosen so as to ensure autonomous operation in the strong periodic self-modulation mode (Figs. 1a, 1b) but away from the developed chaos threshold (the current exceeds the initial value eight and eleven times, respectively). In the first case, we are near the Feigenbaum chaos threshold; the second case corresponds to the situation after the chaos–order transition via intermittency. It is seen that the phase portraits and spectra in these two cases are qualitatively different.

With an increase in the degree of coupling, the oscillations first become quasiperiodic (Fig. 1c) and then chaotic (Fig. 1a) even if coupling is rather weak. The discrete spectral components are distinctly seen against the background of the low noise pedestal. Next, the oscillations become still more irregular, the discrete components decrease and diffuse, while the noise pedestal grows (Fig. 1e). However, at sufficiently high $R$, new discrete peaks appear, this time, at the frequencies contained in the spectrum of the driving signal (Fig. 1f).

Moreover, under certain conditions, self-modulation may again become periodic (for parameter values other than those shown in Fig. 1). Thus, there is an optimal value of $R$ corresponding to the most noisy uniform spectrum.

Note that, in the presence of coupling, both the average output power and the efficiency are higher than those in the case of autonomous operation. This readily follows from the following considerations [12]. In an autonomous oscillator, the distributions of the field and current over the region of interaction are in “antiphase”: near the collector, the beam is well bunched but the field is small; therefore, energy exchange is inefficient. Coupling “corrects” the field structure of the output (driven) tube, making it more uniform and providing more favorable conditions for energy removal from electrons.

If the oscillators are identical ($L_1 = L_2$) and operate near the Feigenbaum chaos threshold, the coupling-induced transition to chaos also follows the Feigenbaum scenario. However, this variant is encountered more rarely than the above-mentioned quasiperiodic route, which is always observed when the oscillators