**VERY HIGH MULTIPLICITY PHYSICS**

Very High Energy Multiplicity Distributions*

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Abstract—Within a geometrical model developed in earlier papers, a change of regime, or a “knee,” is predicted in the multiplicity distributions at large multiplicities. The position and motion of this knee is related to geometrical and KNO scaling and their violation, in particular, the rise of the ratio $\sigma_d/\sigma_t$, as well as to the transition from shadowing to antishadowing, expected at LHC energies. © 2004 MAIK “Nauka/Interperiodica”.

As pointed out in a series of recent papers [1], the dynamics of rare processes with very high multiplicities (VHM) may be quite different from the rest of the events. In this paper, we present our predictions concerning the behavior of the multiplicity distributions $\Psi(z)$ near their high-$z$ border. Our model, in principle, is applicable to any $z$, but here we ignore the shape of the small and moderate $z$, concentrating on the rightmost edge of $\Psi(z)$.

Our knowledge about high-energy multiplicity distributions comes from the data collected at ISR, the $SppS$ collider (UA1, UA2, and UA5 experiments) and the Tevatron collider (CDF and E735 experiments). It should be noted that the recent results from the E735 collaboration taken at the Tevatron [2] do not completely agree with those obtained by the UA5 collaboration at comparable energies at the $SppS$ collider [3].

Note that the delicate features of $\Psi(z)$ at very large multiplicities near the large-$z$ edge can be better seen if the variable $z$ is used instead of $n$.

On the theoretical side, it became common [4–7] to approximate the observed distributions by the convolution of two binomial distributions, accounting for the general bell-like shape of $P(n)$ with the observed structures (knee and possible oscillations) superimposed.

One of the hottest issues in this field is the dynamics of the VHM [8], which is close to the kinematical limit imposed by the phase space. The VHM events are very rare, making up only about $10^{-7}$ of the total cross sections at the LHC energy, which makes their experimental identification very difficult. An intriguing question is the possible existence of a cutoff in the VHM region, beyond $z = n/\langle n \rangle \approx 5$, where $\langle n \rangle$ is the mean multiplicity. In our opinion, a better understanding of the underlying physics can be inferred only in a model involving both elastic and inelastic scattering related by unitarity. Such an approach has been advocated in a series of papers [9–11], summarized in [12].

After a brief summary of the main ideas within this approach, we analyze the relation of the distribution of secondaries and the behavior of the elastic and total cross sections with the possible transition from shadowing to antishadowing [13].

We show that the existence of the cutoff at high multiplicities in the distribution of $\Psi(z)$ is related to the validity of the GS and KNO scaling.

The basic idea of the geometrical approach to the multiple production used in the present paper is that the number of the secondaries at a given impact parameter $\rho$, $n(\rho, s)$ is proportional to the amount of the hadronic matter in the collision or the overlap function $G(\rho, s)$,

$$\langle n(\rho, s) \rangle = N(s)G(\rho, s),$$

where $N(s)$ is related to the mean multiplicity, not specified in this approach, and $G(\rho, s)$ is the overlap function related to the elastic scattering by unitarity,

$$\text{Im} h(\rho, s) = |h(\rho, s)|^2 + G(\rho, s),$$

where $h(\rho, s)$ is the elastic amplitude in the impact parameter representation. Unitarity, a key issue in this approach, enters in the definition of both the elastic amplitude and the inelastic overlap function.

In $u$-matrix unitarization (see [12] and references therein),

$$G_{\text{in}} = \frac{\text{Im} u}{1 + 2\text{Im} u + |u|^2}.$$
where \( u \) is the elastic amplitude (input, or the "Born term").

We use a dipole (DP) model for the elastic scattering amplitude, exhibiting geometrical features and fitting the data. After \( u \)-matrix unitarization, the elastic amplitude reads (see [12])

\[
h(\rho, s) = \frac{u}{1 - iu},
\]

where \( u(y, s) = i e^{-y}, y = \frac{\rho^2}{4 \alpha L}, \) and \( L \equiv \ln s \).

Remarkably, the ratio of the elastic to total cross sections in this model,

\[
\frac{\sigma_{el}}{\sigma_t} = 1 - \frac{g}{(1 + g) \ln(1 + g)},
\]

fixes the (energy-dependent) values of the parameter \( g \). Typical values of \( g \) for several representative energies are quoted in [12].

Rescattering corrections to \( G_{in}(\rho, s) \) here will be accounted for phenomenologically according to the following prescription (see [12] and earlier reference therein):

\[
G_{in}(\rho, s) = |S(\rho, s)|G_{in}(\rho, s),
\]

where \( S(\rho, s) \) is the elastic scattering matrix related to the \( u \) matrix by

\[
S(\rho, s) = \frac{1 + iu(\rho, s)}{1 - iu(\rho, s)}.
\]

This procedure is not unique. For example, it allows the following generalization (see [12] and the earlier reference therein):

\[
G_{in}(\rho, s) = |S(\rho, s)|^\alpha G_{in}(\rho, s),
\]

where \( \alpha \) is a parameter, varying between 0 and 1.

We assume

\[
\langle n(\rho, s) \rangle = N(s)G_{in}^\alpha(\rho, s).
\]

The mean multiplicity \( \langle n(s) \rangle \) is defined as

\[
\langle n(s) \rangle = \frac{N(s)(1 + g)}{g} \times \int_0^g \frac{dx}{(1 + x)^2} \left( \frac{1 + x}{1 - x} \right)^\alpha \frac{x}{(1 + x)^2}.
\]

For the distributions, we have

\[
P(n) = \frac{1 + g}{g} \int_0^g \frac{dx}{(1 + x)^2} \left( \frac{1 + x}{1 - x} \right)^\alpha x(1 - x) + 2\alpha x,
\]

where \( z = n/(n) \).

Since the above integral is nonzero only when the argument of the \( \delta \) function vanishes,

\[
n = N \left( \frac{1 + x}{1 - x} \right)^\alpha \frac{x}{(1 + x)^2},
\]

one gets a remarkable relation

\[
z = \frac{\alpha x}{(1 + x)^2 - \alpha(1 - x)^2}.
\]

To calculate the distribution \( \Psi(z) \), one needs the solution to (14). It can be found explicitly for two extreme cases, namely, \( \alpha = 0 \) and \( \alpha = 1 \). Otherwise, it can be calculated numerically.

The maximal value of \( z \), corresponding to \( x = g \) (\( x \) varies between 0 and \( g \)), can be found as

\[
z_{\text{max}} = \frac{\alpha g}{(1 + g)^2 - \alpha(1 - g)^2}.
\]

It can be seen from (15) that \( z_{\text{max}} \) is a constant if \( g \) is energy independent. In the interval between 53 and 900 GeV, the experimentally observed ratio \( \sigma_{el}/\sigma_t \) varies from 0.174 to 0.225, implying the variation of \( g \) from 0.489 to 0.702, and is uniquely determined by the above ratio. This monotonic increase in \( g(s) \) in turn pushes \( z_{\text{max}}(s) \) outwards, terminating when \( g \) reaches unity (according to [12], this will happen around 10 TeV, i.e., at the future LHC), beyond which the term \( 1 - g \) in (15) will start rising again, pulling \( z_{\text{max}}(s) \) back to smaller values (i.e., \( z_{\text{max}}(s) \) has its own maximum in \( s \) at \( g = 1 \)).

The unusual behavior of \( z_{\text{max}}(s) \) is not the only interesting feature of the present approach. This effect can be related to the behavior of the ratio \( \sigma_{el}/\sigma_t \).