Converging Shock Waves in Porous Media

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Abstract—We have numerically solved several problems related to converging shock waves, including (i) one-dimensional spherical and cylindrical waves with cumulation limited to a ball or cylinder of small radius and (ii) shock-wave flow in a cone-shaped solid target. The passage from a continuous loaded substance to a porous medium in these problems leads to a significant increase in both temperature and pressure in the sample. This character of pressure variation depending on the porosity qualitatively differs from the case of plane waves of constant intensity, for which an increase in the sample porosity under otherwise equal conditions of loading always leads to a decrease in the pressure. © 2004 MAIK “Nauka/Interperiodica”.

Consider a flow by which a shock wave propagates in a substance (continuous or porous). The problem is formulated in a commonly accepted form [1]. The substance behind the wave is assumed to be nonporous and described using equations of the hydrodynamics of continuum and the equations of state, while the porosity is taken into account only by setting the initial density for the shock adiabat. Based on the laws of conservation, this approach provides a rather satisfactory description of experimental data on the compressibility of porous substances in many cases, although it fails to explain the results of some experiments showing anomalously high temperature behind the wave front [2].

A question naturally arises as to how the flow parameters vary behind the shock wave front as a result of the change in porosity when the other parameters of the problem remain unchanged. As is well known, the temperature behind the shock wave front increases with the degree of porosity. As for the pressure, the situation is not as evident. The parameters of a shock wave appearing in a porous substance are determined by solving the problem of a decay of discontinuity at the boundary with a medium from which the wave entered the substance. Omitting the details of an analysis of this problem, we can summarize the results as follows.

The derivative of pressure behind the shock wave front with respect to the density \( \rho_{00} \) before the front is positive, \( \partial p / \partial \rho_{00} > 0 \), if two conditions are satisfied: (i) \( du / dp > 0 \) (\( u \) is the velocity behind the front) and (ii) the thermodynamic derivative behind the front is also positive, \( (\partial e / \partial v)_{p} > 0 \) (\( e \) is the specific internal energy, \( v = \rho^{-1} \) is the specific volume). Evidently, both these conditions are always satisfied. Therefore, an increase in the degree of porosity (i.e., a decrease in \( \rho_{00} \)) is practically always accompanied by a decrease in intensity of a shock wave entering into a substance. However, this by no means implies that the pressure behind the shock wave front propagating in a porous substance cannot exceed the value observed for a less porous or continuous medium.

Below, we present a numerical solution to the problem involving convergent shock waves in the presence of a certain mechanism of limitation of the cumulation. Under these conditions, the passage from continuous to porous medium leads to an increase in both temperature and pressure.

In our numerical calculation, the porous substance was graphite with as density of \( \rho_{00} = 1.7 \) g/cm\(^3\) (the crystal density being \( \rho_{0} = 2.26 \) g/cm\(^3\)). The equation of state for graphite is analogous to that used in [3], but the phase transition of graphite into diamond was not taken into account in this calculation.

The problem is formulated using a system of equations of nondissipative hydrodynamics of compressible media,

\[
\begin{align*}
\frac{dp}{dt} + \rho \text{div} \mathbf{u} &= 0, \\
\rho \frac{d\mathbf{u}}{dt} + \mathbf{v} p &= 0, \\
\rho \frac{d(e + u^2/2)}{dt} + \text{div} p \mathbf{u} &= 0,
\end{align*}
\]

closed by the equations of state,

\[
p = p(\rho, T), \quad e = e(\rho, T).
\]
The crystal density $\rho_0 > \rho_{00}$. It is assumed that the porous substance begins to move after passage of the bow shock wave. The boundary condition on the wave is determined by the shock adiabat with the initial state parameters $\rho_{00}, p_0 = p(\rho_{00}, T_0)$, and $\varepsilon_0 = \varepsilon(\rho_{00}, T_0)$.

The problem was numerically solved using two methods of calculation through the bow shock wave. The first method is based on a simple model of pore kinetics (see, e.g., [4]). The second method is based on the model [3] of a two-phase (graphite–ideal gas) mixture. The results of control calculations using both methods showed good mutual agreement.

Consider a spherical shock wave in a graphite ball with a radius of 2 mm caused by the impact of an aluminum ball impinging at a velocity of 1 km/s. The calculations were performed to within the second order of accuracy on a Lagrange lattice with 100, 200, 400, or 800 intervals in graphite, which showed evidence of convergence of the numerical solution.

Figure 1a shows the calculated plots of the pressure $p$ at the wave front versus the front radius $r$. Beginning with a certain radius $r$, the pressure in the porous graphite is significantly higher than that in the continuous graphite. Introducing limitations on the cumulation in the form of a ball of small radius $r_0$ [5], we obtain an analogous relationship for the maximum pressures in the system. This is illustrated in Fig. 1b, which shows the plots of maximum pressure versus time for three values of $r_0$. As can be seen, the passage from continuous to porous graphite leads to an almost twofold increase in the maximum pressure. For an analogous cylindrical wave, the maximum pressure increases upon passage from continuous to porous graphite by a factor of about 1.5.

Let us consider the shock compression of graphite in the case of cone-shaped solid targets [3]. The impinging body strikes a steel target containing a cone-shaped cavity filled with graphite. In this case, Eqs. (1) were solved using a scheme described in [3]. The main calculations were performed on a $25 \times 50$ lattice (the first is the number of intervals along the cone base and the second is that along the axis of symmetry). The control calculations were performed on a lattice of double density in both directions.

Figure 2 shows the families of isobars for the initial porous graphite for two successive moments of time, calculated using the following parameters: aluminum ball striker velocity, 2.5 km/s; cone angle, 75°; cone base radius, 2.5 mm; steel spacer width between graphite cone and striker, 8 mm. Figure 2a depicts the bow shock wave near the axis of symmetry and the pulse of compression from the side boundary of the cone. Exact calculation of the bow shock wave parameters based on the conservation laws gives $\sim 9$ GPa for the pressure behind the front. Numerical calculation gives approximately the same value, which is evidence of the adequacy of the employed scheme. In the case of continuous graphite, the pressure behind the shock wave front increases up to $\sim 17$ GPa. By the next time instant illustrated in Fig. 2b, a shock wave of nearly spherical shape is formed that converges to the cone vertex. The mechanism of limited cumulation in this wave corresponds to deformation of the cone wall.