SH-Wave Intromission Concept

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\textit{Abstract}—The existence of an SH-wave incidence angle for which the reflected amplitude is zero (SH-wave intromission angle) is established for the case of plane-wave scattering by a planar interface joining two homogeneous, isotropic, and linearly elastic media. Such an incidence angle is numerically shown to exist for two combinations of bimaterial interface properties. The SH-wave intromission angle is roughly parallel to the electromagnetic Brewster angle and the acoustic P-wave intromission angle, and the concept should find new applications for non-intrusive characterization of interfaces. © 2004 MAIK “Nauka/Interperiodica”.

\textbf{Introduction.} Three types of elastic body waves composed of longitudinal primary (P) and transverse secondary (SV and SH) waves can exist in homogeneous, isotropic, and linearly elastic media. P-waves are polarized in the propagation direction and within the incidence plane, SV-waves are polarized orthogonal to the propagation direction and within the incidence plane, and SH-waves are polarized orthogonal to the propagation direction and within a plane that is orthogonal to the incidence plane. When elastic waves are incident on an interface joining media with different impedances, scattering (reflection and refraction/transmission) occurs. Amplitude and energy partitioning between scattered components is dependent on the incident wave type, the incidence angle, and the media impedances.

Amplitudes and energies of planar interface-scattered components from incident P- and SV-waves depend on media compressional velocities, shear velocities, and densities, whereas partitioning from incident SH-waves depends only on media shear velocities and densities [1]. For incident P- or SV-waves, mode-conversion can occur at the point of oblique incidence on a welded planar interface, with four possible wave types (reflected P, reflected SV, refracted P, and refracted SV) being generated. From an interface parallel to incident SH-wave polarization however, only scattered SH-waves are generated, regardless of the incidence angle. We have derived equations from the plane SH-wave reflection coefficient, which predict an angle of SH-wave incidence at which the reflected amplitude is zero (SH-wave intromission angle).

\textbf{Theory.} For plane SH-waves in homogenous, isotropic, and linearly elastic media, the reflection coefficient [1] for any angle of incidence can be written as

\[ SH_2/SH_1 = (Z_s \cos \theta_{SH,SH} - Z_s \cos \theta_{SH,SH})/(Z_s \cos \theta_{SH,SH} + Z_s \cos \theta_{SH,SH}), \]

where \( SH_2/SH_1 \) is the reflection displacement amplitude coefficient, \( \theta_{SH,SH} \) is the incidence and reflection angle, \( \theta_{SH,SH} \) is the refraction angle, and \( Z_s \) and \( Z_s \) are the incident (medium 1) and refracted (medium 2) media shear impedances. Shear impedance is the product of medium shear velocity (\( V_s \) or \( V_s \)) and density (\( \rho_1 \) or \( \rho_2 \)).

The reflection coefficient is zero (only the refracted wave will remain) when the incidence angle is equal to the SH-wave intromission angle (\( \theta_1 \)), and this occurs when

\[ Z_s \cos \theta_1 = Z_s \cos \theta_{SH,SH}, \]

To solve for \( \theta_1 \),

\[ Z_s/Z_s = \cos \theta_{SH,SH} / \cos \theta_1. \]

Using the relation \( \sin^2 x + \cos^2 x = 1 \) and Snell’s law [1] to eliminate \( \theta_{SH,SH} \),

\[ Z_s/Z_s = (1 - (V_s/V_s)^2 \sin^2 \theta_1)^{1/2}/(1 - \sin^2 \theta_1)^{1/2} \]

yields a solution for \( \theta_1 \) in terms of media shear velocities and shear impedances,

\[ \theta_1 = \sin^{-1} (1 - (Z_s/Z_s)^2/(V_s/V_s)^2 - (Z_s/Z_s)^2)^{1/2} \]

or a solution for \( \theta_1 \) in terms of media shear velocities.
SH-waves and densities,
\[ \theta_1 = \sin^{-1} \left( \frac{(V_{S2}/V_{S1})^2}{-\left(\rho_1/\rho_2\right)^2/(V_{S2}/V_{S1})^4 - \left(\rho_1/\rho_2\right)^2} \right)^{1/2}, \]
where \( \theta_1 \) is a particular real angle of incidence between zero and 90°. To obtain a real solution to Eq. (5) or (6), the numerator and denominator in the respective equation must have the same sign, and the absolute value of the numerator must be equivalent to or less than the absolute value of the denominator in the respective equation. In terms of media shear velocities and shear impedances, Eq. (5) shows that these conditions will be met if \((V_{S2}/V_{S1}) > 1 > (Z_{S1}/Z_{S2})\), or if \((Z_{S1}/Z_{S2}) > 1 > (V_{S2}/V_{S1})\). When both \(V_{S1} = V_{S2}\) and \(\rho_1 = \rho_2\), the SH-wave reflection coefficient is zero for all incidence angles.

**Numerical examples.** To illustrate SH-wave intromission angle existence, solutions to equations (obtained using the PSHSV computer program [2]) describing plane SH-wave scattering from a planar interface joining two homogeneous, isotropic, and linearly elastic half-spaces are presented. Shown in Figs. 1 and 2 are solutions for two example bimaterial interface physical properties cases. In Fig. 1 are solutions for the case of an incident SH-wave going from a medium with relatively low shear velocity to a medium with relatively high shear velocity. In Fig. 2 are solutions for the case of an incident SH-wave going from relatively high to relatively low shear velocity. Shear velocity (m/s) and density (g/cm³) values used to obtain solutions are listed on the plots in Figs. 1 and 2.

Both Fig. 1 and Fig. 2 each contain six subplots showing how amplitude coefficients, square root energy coefficients, energy coefficients, and phase angles change as a function of incidence angle for each wave type generated at the interface for the respective case. The amplitude coefficients plotted in Figs. 1 and 2 were calculated using Eq. (1), and the following equation,
\[ SH_1SH_2 = (2Z_{S1}\cos\theta_{SH_1SH_1}Z_{S1}\cos\theta_{SH_1SH_1})/(Z_{S1}\cos\theta_{SH_1SH_1} + Z_{S2}\cos\theta_{SH_2SH_2}), \]
where \(SH_1SH_2\) is the refraction displacement amplitude coefficient. The reflection and refraction angle cosines in Eqs. (1) and (7) depend on the horizontal and vertical components of the slowness vector,
\[ pp = (u^2 - \eta^2)^{1/2} = u \sin\theta, \]
\[ \eta = (u^2 - pp^2)^{1/2} = u \cos\theta, \]