INTRODUCTION

In a classical scheme, fulfillment of the 2:1 betatron relation is realized owing to magnetic gaps in the interpole space and in the central part (disk-coil unit), which ensure the required relationship between the magnetic-field inductions at the radius of the equilibrium orbit and within the circle of the orbit that are created by the magnetization coil [1, 2]. With increasing radius, the magnetic-field energy grows according to a quadratic law, while the kinetic energy of electrons is proportional to the radius; that is,

\[ E_k = \langle \beta \rangle cBR_0, \quad \langle \beta \rangle = \int_1^\gamma \frac{1}{\gamma^2} d\gamma, \]  

where \( \langle \beta \rangle \) is a mean relative velocity, \( \gamma = 1 + cE/Q/c0 \) is a relativistic factor, \( c \) is the speed of light, and \( B \) is the magnetic-field induction at the equilibrium radius \( R_0 \).

With respect to the energy of an oscillatory circuit, the electromagnet weight, and the kinetic energy, the most elaborate constructions of betatrons—a small-size betatron with \( R_0 = 6 \) cm and a high-current betatron with \( R_0 = 30 \) cm [2], which are characterized by the same ratio of the chamber height to the radius, \( H/R_0 = 0.7 \), and by identical values of the magnetic-field induction in steel—are in the ratios \( 6/30^2 \sim 120 \) J/3 \( \times 10^3 \) J = 90 kg/2.7 \( \times 10^3 \) kg = 6 MeV/30 MeV.

There are many approaches aimed at reducing the electromagnet weight owing to the demagnetization of steel and an increase in the magnetic-field-induction variations in it. However, a practical implementation of these approaches in [5] for a model of a 300-MeV betatron revealed that acceleration is unstable from one cycle to another and that the resulting intensity proves to be much lower than its calculated counterpart. A simpler scheme of the demagnetization of an electromagnet was proposed in [6]. This scheme ensured the calculated acceleration of a charge, but it required a considerable interval between pulses in order to demagnetize the electromagnet core. In the classical scheme featuring air gaps, the nonlinearity of the hysteresis loop has virtually no effect on the behavior of the radius of the equilibrium orbit at the beginning of the acceleration cycle, but, in the case of a closed magnetic circuit, it requires the application of dedicated correction circuits. For this, use was made of a saturable-core choke coil (weight about 20% of the magnet weight) in [5] and of an additional circuit with an independent power-supply unit in [6, 7]. The connection of windings in parallel and, as a consequence, the appearance of equating currents in response to a change in the \( Q \) factor of the windings because of heating, as well as the requirement of a high precision in synchronizing the operation of high-voltage power supply units, reduced the effect of harnessing the demagnetization of steel and hindered its application in practice.

1. ENERGY RELATIONS IN A LEAKAGE-FLUX BETATRON

The leakage flux in a transformer is determined by the loading current and is localized in the air gap between the primary and the secondary winding. The strength of the leakage-flux magnetic field is governed by the magnetization current in the primary winding. Together with the electric-field strength, this magnetic-field strength forms the Poynting vector [8] responsible for energy transfer from the primary to the secondary circuit.

An electron beam in the orbit in the electromagnetic field of a betatron plays the role of a winding and a load simultaneously and interacts with the energy flux coming from the capacitance storage device in a magnetic...
The electric-field strength is determined by the derivative of the magnetic flux and is therefore independent of its absolute value; it follows that part of the increment of the magnetic flux in the central region of the orbit can be changed from negative to positive values of the magnetic induction, and this circumstance is used in a leakage-flux betatron [9, 10] (see Fig. 1).

In the initial state, the thyristors \( T_1, T_3, \) and \( T_4 \) are switched on and the capacitance storage device \( C_0 \) and the capacitor \( C \) are preliminarily charged through the choke coil from the rectifier \( B \). In the steady-state regime (see Fig. 2), the magnetic state of the magnetic circuit is determined by the magnetic flux of the winding \( W_1 \) carrying the current \( I_0 \) and is given by

\[-B = B_1 - L_1 I_0 / W_1 S_c,\]  

where \( L_1 \) is the inductance of the winding on a closed magnetic circuit, \( B_1 \) is the residual magnetic-field induction in the steel of the magnetic circuit with allowance for the technological gap \( \Delta \), and \( S_c \) is the cross-sectional area of steel in the segment being considered.

In Fig. 1c, the initial magnetic state of the core is characterized by point \( I \) and the magnetic-field induction \( B \). The thyristors \( T_1, T_2, \) and \( T_3 \) are switched on at the instant \( t_1 \). The current discharging the capacitance storage device \( C_0 \) flows in the windings \( W_1 \) and \( W_2 \) connected in series and oppositely, and the formation of the magnetic fields \( B_1(t) \) and \( B_0(t) \) occurs there (see Fig. 2). As to the magnetic field in the interpole gap (leakage flux), it is determined by the ampere-turns of the winding \( W_2 \) and is dependent on the height \( H \) of the interpole space, so that

\[B_0(t) = \frac{\mu_0 I_2(t) W_2 K}{H},\]  

where

\[I_2(t) = \frac{V_e}{R_0 + C_0/\omega},\]  

\[V_e = -\frac{\Delta B_1}{\Delta t},\]  

\[\Delta B_1 = B_{1f} - B_{1i},\]  

\[\omega = \sqrt{L_2 / C_0},\]  

\[R_0 = \frac{L_2}{C_0},\]  

\[I_1 = \frac{V_e}{R_0 + L_2/\omega},\]  

\[B_{1i} = \frac{V_e}{R_0 + L_2/\omega} \]  

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In Fig. 1c, the initial magnetic state of the core is characterized by point \( I \) and the magnetic-field induction \( B \). The thyristors \( T_1, T_2, \) and \( T_3 \) are switched on at the instant \( t_1 \). The current discharging the capacitance storage device \( C_0 \) flows in the windings \( W_1 \) and \( W_2 \) connected in series and oppositely, and the formation of the magnetic fields \( B_1(t) \) and \( B_0(t) \) occurs there (see Fig. 2). As to the magnetic field in the interpole gap (leakage flux), it is determined by the ampere-turns of the winding \( W_2 \) and is dependent on the height \( H \) of the interpole space, so that

\[B_0(t) = \frac{\mu_0 I_2(t) W_2 K}{H},\]  

where

\[I_2(t) = \frac{V_e}{R_0 + C_0/\omega},\]  

\[V_e = -\frac{\Delta B_1}{\Delta t},\]  

\[\Delta B_1 = B_{1f} - B_{1i},\]  

\[\omega = \sqrt{L_2 / C_0},\]  

\[R_0 = \frac{L_2}{C_0},\]  

\[I_1 = \frac{V_e}{R_0 + L_2/\omega},\]  

\[B_{1i} = \frac{V_e}{R_0 + L_2/\omega} \]  

\[L_2 = \frac{I_1^2}{C_0},\]  

\[C_0 = \frac{V_e}{\omega I_1},\]  

\[V_e = V_{in} - V_{out},\]  

\[I_1 = I_0 - I_2(t),\]  

\[I_2(t) = \frac{V_e}{R_0 + L_2/\omega},\]  

\[V_e = \Delta B_1 / \Delta t,\]  

\[\Delta B_1 = B_{1f} - B_{1i},\]  

\[\omega = \sqrt{L_2 / C_0},\]  

\[R_0 = \frac{L_2}{C_0},\]  

\[I_1 = \frac{V_e}{R_0 + L_2/\omega},\]  

\[B_{1i} = \frac{V_e}{R_0 + L_2/\omega} \]  

\[L_2 = \frac{I_1^2}{C_0},\]  

\[C_0 = \frac{V_e}{\omega I_1},\]  

\[V_e = V_{in} - V_{out},\]  

\[I_1 = I_0 - I_2(t),\]  

\[I_2(t) = \frac{V_e}{R_0 + L_2/\omega},\]  

\[V_e = \Delta B_1 / \Delta t,\]  

\[\Delta B_1 = B_{1f} - B_{1i},\]  

\[\omega = \sqrt{L_2 / C_0},\]  

\[R_0 = \frac{L_2}{C_0},\]  

\[I_1 = \frac{V_e}{R_0 + L_2/\omega},\]  

\[B_{1i} = \frac{V_e}{R_0 + L_2/\omega} \]  

Fig. 1. (a) Electromagnet of a leakage-flux betatron: \( W_1, W_2 \) magnetization and compensation windings, \( I \) central core, \( 2 \) backward magnetic circuit, \( 3 \) poles, \( 4 \) injector, and \( 5 \) contour of the vacuum chamber; (b) circuit of the betatron power-supply unit: \( B \) rectifier, \( C_0 \) capacitance storage device, \( T_1, T_2, D_1, D_2 \) thyristors and diodes of the high-voltage circuit, \( T_3 \) thyristor of the \( R, C_k, R_k \) circuit of equilibrium-radius correction, \( T \) discharge thyristor, \( C_0 \) capacitor and inductance choke of the filter, and \( T_k \) thyristor of the stabilization and control of energy; and (c) magnetic characteristics of the electromagnet and the air gap: \( B_1(H), B_0(H) \) dependence of the magnetic-field induction in steel and in the air gap on the magnetic-field strength and \( 1–6 \) characteristic points on the magnetic curves matched in time with the values of the current in the windings (Fig. 2).