Abstract—This paper completes a series of reviews devoted to the physics of complex plasmas, in which one of the components (dust) is in a crystalline or liquid state, while the others (electron, ions, and neutral atoms) are in a gaseous state. This review is devoted to the theoretical approaches used to describe complex plasmas so far. The main theoretical developments have been concentrated in the gaseous and weakly nonideal states of complex plasmas. Here, we describe the achievements in the new kinetic and new hydrodynamic approaches to complex plasmas. At present, only generalizations of the van der Waals approach for complex plasmas have been used to describe phase transitions and plasma condensation in complex plasmas. Here, criteria for transitions are described and compared with the existing experimental observations. Theoretical and numerical results for nonlinear structures, such as dust layers, dust voids, dust sheaths, and dust convective vortices, obtained by solving the stationary balance equations, are also discussed and compared with state-of-the-art experiments. At present, experiments in this field are progressing very fast, while theory is not advancing at the same rate of development. To further develop new theoretical models, one can use the elementary physical processes in complex plasmas described in the previous parts of the review. However, the detailed comparison of theory and experiments also needs more detailed experimental diagnostics of the phenomena observed. In the concluding part of our review, the trends in experiment and theory, as well as some existing applications, including industrial, environmental, and astrophysical ones, are described. © 2004 MAIK “Nauka/Interperiodica”.

1. THEORETICAL APPROACHES IN COMPLEX PLASMAS

1.1. Introduction

In the previous parts of our review, we described in detail the processes of dust charging in complex plasmas [1]; the elementary processes in complex plasmas [2], including the external forces that act on dust grains and dust–dust interactions; and state-of-the-art experiments [3].

We recall here the notation used in the previous parts of our review and used in this part as well. The quantity \( P = n_d Z_d / n_i \) is the so-called Havnes parameter, showing the relative number of charges on the dust grains (here, \( n_d \) is the dust density, \( n_i \) is the ion density, and \( Z_d \) is the dust charge in units of elementary charge). In state-of-the-art experiments, the parameter \( P \) (which is always less than unity) is on the order of unity (the minimum value of \( P \) in state-of-the-art experiments is \( 0.5 \times 10^{-2} \)). The dimensionless ion and electron densities are denoted as \( n = n_i / n_{i,0} \) and \( n_e = n_e / n_{e,0} \) (where \( n_{i,0} \) is a certain characteristic ion density, or density far from the structures or the density corresponding the basic state). The parameters of which are determined by both the charge and power balance (see [2])). The small parameter \( \tau = T_i / T_e \) is on the order of \( 10^{-2} \) in state-of-the-art experiments. The sharpness of a dust structure boundary is determined by another (much smaller) parameter \( \tau_d = T_i / T_e \) (where \( a \) is the dust size and \( T_i \) is the dust temperature). In state-of-the-art experiments, \( \tau_d \) is no larger than \( 2 \times 10^{-3} \), but it is usually on the order of \( 10^{-6} \). The ion Debye radius is determined by the expression \( \lambda_{DI} \).

To estimate the contributions from the various processes, we used in [3] expressions for the elementary processes in complex plasmas given in [2]. For simplicity, these processes were described by some average quantities, such as the electron, ion, and dust densities; the temperatures; and the ion drift velocity. The description of elementary processes in this form is very useful in applications. However, even in such a description, some processes are determined by the particle velocity distributions. For example, this is true for the charging currents [1], the ion drag force, and the dust–dust interactions [2], which depend strongly on the ion velocity distribution. For simplicity and to be able to perform some estimates, we have already given (see [2]) the results for the case of thermal distributions of electrons and ions. More general is a kinetic approach

\[1\] This article was submitted by the authors in English.
using velocity distribution functions for all of the components. This requires the use of a kinetic description of complex plasmas. A general kinetic theory of complex plasma should be more sophisticated than the standard plasma kinetic theory because of the openness of the system and the dust charge variability (see [4–8]). In general, even the foundations of the kinetic approach should be changed in complex plasmas [4]. We start here with showing the necessity of kinetic description using simple estimates for the thermalization time of the particle distributions, and will show that the kinetic description is indeed dictated by the parameters of the state-of-the-art experiments. We shall then recall briefly how the kinetic description is formulated in ordinary plasmas and emphasize the important role of fluctuations in such a description. It becomes obvious that, to formulate a basic description in the kinetic theory of complex plasmas, it is necessary to develop a more complicated description of fluctuations in complex plasmas as compared to that in ordinary plasmas. In particular, these fluctuations should take into account the variability of dust charges, the openness of the system, and the presence of external sources. The basic kinetic description in ordinary plasma theory is based on deriving equations containing collision integrals. The same needs to be performed for complex plasmas; above all, generalized collision integrals must be found. Thus, the ordinary kinetic approach cannot be used for complex plasmas and new kinetic equations must be found as the basic equations for the kinetic description of complex plasmas. This is one of the major problems in the theory of complex plasmas; the solving of this problem will lay the foundation for other methods for describing complex plasmas. If we take the velocity momenta of these equations, a hydrodynamic equation should be found. The hydrodynamic description of complex plasma will thus be based on a new set of equations. These, in turn, can be used for other theoretical models and for the numerical simulation of dust structures. The first steps toward the foundations of the new kinetic description of complex plasmas with several simplifying assumptions (see below) were made in [4–8], and the first hydrodynamic description with the elementary processes in complex plasmas taken into account was made in [9]. In [9], the new hydrodynamic equations were not derived from the general kinetic equations. This was performed in [8] and led both to the generalization of the coefficients used in [9] and to the description of new effects related to heat transport.

1.2. The Kinetic Approach in Complex Plasmas

1.2.1. Estimates for the thermalization of ion and electron distributions. Before discussing the first results of the theoretical approach to the kinetic description of complex plasmas, let us make some simple estimates of the thermalization processes for the parameters of the existing laboratory experiments and try to answer the following questions:

(i) Is changing the foundations of the kinetic theory only a superficial exercise in which one can use simple considerations for practical applications, or is an appropriate theoretical consideration dictated by the experiments?

(ii) Is it indeed necessary to reformulate the basic kinetic approach?

The first question can be reformulated as follows: Are the plasma particle distributions indeed thermal in state-of-the-art experiments, or can they be nonthermal? Is it possible to answer the question as to whether the particles are able to form a thermal distribution under experimental conditions when direct measurements are not performed? If the distributions are not thermal, then all of the coefficients describing the elementary processes should be modified and their value for the thermal distribution can serve only as a very rough estimate. The need for the development of the kinetic approach in complex plasmas then becomes obvious.

At this point, it should be remembered that complex plasmas have several components: electrons, ions, neutral atoms, and dust. Estimates will be different for different components. We can expect that the dust component should more often be found to be thermal due to strong dust interactions. For Coulomb dust–dust collisions, this is, in a certain sense, unimportant. Generally, however, when screening and collective interactions are taken into account, the theoretical problems of whether the dust can be distributed as thermal and what is the time scale of dust thermalization should be resolved. In some expressions for the processes in complex plasmas, there are integrals over the velocity of the electron and ion distribution functions and these integrals are typically only slightly sensitive to the details of these distribution functions (for example, the numerical coefficients change by a factor on the order of unity). However, we have examples [2] where, in some cases, these integrals can be very sensitive to the ion distribution function (as is the case with the presence of ion drift). If the particle distribution is not thermal, the estimates of the criteria of transition to a strongly correlated state using thermal distributions can be considered only as a rough first approach to get some intuitive conclusions for the processes determining this transition. In principle, the kinetic approach provides not only a detailed description of the transition to a strongly correlated state but can be checked experimentally on the kinetic level, since any dust grain motion can be detected during the transition. Such a full kinetic description that includes the transition to a strongly correlated state of a complex plasma does not as yet exist.

The first steps toward the description of the gaseous states of complex systems were made in [4–8] and in [10, 11]. The gaseous states will have many unusual kinetic features that can precede the conversion into a strongly correlated state. These can be investigated very simply on the kinetic level. The approaches of [4–8]