Quasistatic Waves of Hydrogravity Generated in the Galactic Interstellar Medium by a Pulsating Neutron Star

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Received February 25, 2004

Abstract—Based on principles of classical hydrodynamics and Newtonian gravity, the theory of hydrogravity, formulated in the manner of hydromagnetic theory, is developed to account for the gravitational effect of global pulsations of a star on the motions of the ambient gas–dust interstellar medium. Analytic derivation of the dispersion relation for canonical gravity waves at the free surface of an incompressible viscous liquid is presented, illustrating practical usefulness of the proposed approach, heavily relying on the concept of classical gravitational stress introduced long ago by Fock and Chandrasekhar, and accentuating the shear character of this mode. Particular attention is given to gas-dynamical oscillations of a similar physical nature generated by a pulsating neutron star in an unbounded spherical shell of gas and dust promoted by circumstellar gravitational stresses and damped by viscosity of the interstellar matter. Computed in the long-wavelength approximation, the periods of these gravity-driven shear modes, referred to as quasistatic modes of hydrogravity, are found to be proportional to periods of the gravity modes in the neutron star bulk. Given that collective oscillations of cosmic plasma in the wave under consideration should be accompanied by electromagnetic radiation and taking into account that only the radio waves of this radiation can freely travel through the galactic gas–dust clouds, it is conjectured that the considered effect of gravitational coupling between seismic vibrations of a neutron star and fluctuations of the galactic interstellar medium should manifest itself in the radio range of pulsar spectra. Some useful implications of the theory developed here to a number of current problems of asteroseismology are briefly discussed. © 2004 MAIK “Nauka/Interperiodica”.

1. INTRODUCTION

It was realized long ago that the restless behavior of neutron stars, exhibited in pulsar spectra by millisecond micropulses, owes its origin to seismic vibrations triggered either by implosion effects of supernova events or by starquakes \cite{1–3}, which may be connected with some short gamma-ray bursts \cite{4, 5}. At present, there are tolerably coherent arguments showing that neutron stars (both pulsars and magnetars) can support long-lasting pulsations driven by bulk forces of elasticity, gravity, and magnetism of neutron star matter \cite{6–10}. At the same time, the influence of neutron star pulsations on a gas–dust interstellar medium (ISM), which serves as a fluid matrix mediating a vast variety of gas-dynamical processes (e.g., \cite{11}), remains less studied. This work discusses the hydrodynamic mechanism of gravitational coupling between seismic vibrations of a neutron star and fluctuations of gas–dust flows in the ambient envelope. Specifically, we consider a model in which a pulsating neutron star embedded in a gas–dust spherical shell is regarded as a source of large-scale hydrodynamical wave motions promoted by circumstellar gravitational stresses and damped by viscous stresses. The oscillatory motions in question have the same physical nature as the gravity waves at the free surface of an incompressible viscous fluid caused by the presence of a constant field of Newtonian gravity (e.g., \cite{12–14}), the wave process being well known in the physics of planetary atmospheres \cite{15}.

This paper presents arguments that proper mathematical treatment of these gravity-driven wave motions of the interstellar medium can be developed on the basis of self-consistent equations for variables of classical hydrodynamics and Newtonian gravity, which are formulated in a manner of equations that govern the hydromagnetic theory. In pursuit of this aim, we follow two different approaches, both relying on the key concept of Newtonian gravitational stress. The underlying

\footnote{This article was submitted by the authors in English}
idea of the first method, constituting the content of Sections 2 and 3, is to include the Newtonian gravitational field in a set of gas-dynamical variables of circumstellar motions by considering this field on an equal footing with the standard hydrodynamical variables such as density and velocity. The second method, formulated in the Appendix, is based on coupled equations involving density, the velocity, and the gravitational stress tensor. Particular attention is drawn to the fact that both these methods yield analytically identical estimates for the existence to fluctuations in circumstellar gravitational frequency and lifetime of the gravity modes owing their methods. In the discussion, we point out some useful applications of the theory developed here.

2. GOVERNING EQUATIONS OF HYDROGRAVITY

The point of departure in our considerations is the representation of the body force of gravity

\[ \mathbf{F} = -\rho \mathbf{g}, \quad \nabla \mathbf{g} = 4\pi G \mathbf{p}, \quad \mathbf{g} = -\nabla U \]  

via the tensor of gravitational stresses \( G_{ik} \):

\[ F_i = -\rho g_i = -\frac{\partial G_{ik}}{\partial x_k}, \]

\[ G_{ik} = \frac{1}{4\pi G} \left[ g_{ik} - \frac{1}{2} (g_j g_j) \delta_{ik} \right]. \]  

To the best of our knowledge, this form of the gravitational force in the stationary material continuum of density \( \rho \) was first discussed long ago by Fock [17] and justified by Chandrasekhar [18]. Such a possibility is apparent from the identity

\[ F_i = \rho \frac{\partial U}{\partial x_i} = -\frac{\partial}{\partial x_k} \left\{ \frac{1}{4\pi G} \left[ \frac{\partial U}{\partial x_i} \frac{\partial U}{\partial x_k} - \frac{1}{2} \left( \frac{\partial U}{\partial x_j} \frac{\partial U}{\partial x_i} \right) \delta_{ik} \right] \right\}. \]  

Also, the discussion of Newtonian gravitational stresses can be found in [19, 20]. A matter of particular interest for our present discussion is Chandrasekhar’s suggestion [18] that the above tensor representation for the static force of Newtonian gravity be incorporated in the dynamical description of the gravity-driven motions of an inviscid fluid. Specifically, it is shown in [18] that replacement of the standard expression for the gravitational force,

\[ F_i = -\rho g_i, \]

in the Euler equation for an ideal fluid

\[ \rho \frac{dV_i}{dt} = -\frac{\partial P}{\partial x_i} - \rho g_i, \quad \frac{d}{dt} + V_j \frac{\partial}{\partial x_j}, \]  

by the above tensor representation

\[ F_i = -\nabla_k G_{ik} \]

allows one to rewrite (4) in the form of a conservation law for the density of linear momentum \( \rho V_i \),

\[ \frac{\partial (\rho V_i)}{\partial t} = -\frac{\partial P_i}{\partial x_k} = \rho V_j V_k + P \delta_{ik} + G_{ik}, \]  

where \( P_{ik} \) is the total flux density.

We recall at this point that the key statement in the MHD theory is that the state of motion of a magnetohydrodynamically conducting ISM threaded by a galactic magnetic field \( B \) can be uniquely specified by the density \( \rho(\mathbf{r}, t) \), the flow velocity \( \mathbf{V}(\mathbf{r}, t) \), and the magnetic flux density \( \mathbf{B}(\mathbf{r}, t) \), which are regarded on an equal footing as independent dynamical variables (see, e.g., [21–23]). The equations of dissipation-free MHD theory

\[ \frac{\partial \rho}{\partial t} = -\frac{\partial \rho \mathbf{V}_i}{\partial x_k} \]

\[ \rho \frac{dV_i}{dt} = -\frac{\partial}{\partial x_k} \left\{ \rho \mathbf{V}_j \mathbf{V}_k \mathbf{V}_i \mathbf{B}_j \mathbf{B}_k \right\} \]

\[ \frac{\partial B_i}{\partial t} = \frac{\partial}{\partial x_k} \left\{ \mathbf{V}_j \mathbf{B}_k \mathbf{B}_i \mathbf{B}_j \right\} \]

describe the fluid mechanics of a highly ionized (perfectly conducting) ISM threaded by a galactic magnetic field \( B \).

Remarkably, Eq. (5) permits the equivalent representation

\[ \rho \frac{dV_i}{dt} = -\frac{\partial}{\partial x_k} \left\{ \rho \mathbf{V}_j \mathbf{V}_k \mathbf{V}_i \mathbf{B}_j \mathbf{B}_k \right\} \]

\[ \rho \frac{dV_j}{dt} = -\frac{\partial}{\partial x_k} \left\{ \rho \mathbf{V}_j \mathbf{V}_k \mathbf{V}_i \mathbf{B}_j \mathbf{B}_k \right\} \]

which in appearance is similar to the Euler equation of the hydromagnetic model for interstellar gas dynamics. This then indicates that the constructive treatment of the gravity-driven gas dynamics of the interstellar medium can be developed on a methodological footing similar to that lying at the base of magnetohydrodynamics. In particular, this suggests that the gravitational field \( \mathbf{g}(\mathbf{r}, t) \) can be regarded as an independent variable of the ISM motion on an equal footing with basic variables of interstellar gas dynamics, the density \( \rho(\mathbf{r}, t) \) and the flow velocity \( \mathbf{V}(\mathbf{r}, t) \). Then, adhering to this point of view, our next goal is to specify the form of the constitutive equation for the gravity-flow coupling, that is, an equation describing the kinematic relation between the vector field of classical gravity \( \mathbf{g}(\mathbf{r}, t) \) and the density of linear momentum \( \rho(\mathbf{r}, t) \mathbf{V}(\mathbf{r}, t) \).