1. INTRODUCTION

According to a presently popular idea, our observable Universe can be located on a four-dimensional surface, called the brane, embedded in a higher-dimensional manifold, called the bulk. This “braneworld” concept, suggested in the 1980s [1], is broadly discussed nowadays, mainly in connection with the recent developments in supersymmetric string/M-theories [2]. One reason why we do not see any extra dimensions is that the observed matter is confined to the brane and only gravity propagates in the bulk. There are numerous applications of the braneworld concept to particle physics, astrophysics, and cosmology, such as the hierarchy problem and the description of dark matter and dark energy [3].

Most of the studies are restricted to infinitely thin branes with deltlike localization of matter. A well-known example is Randall and Sundrum’s second model (RS2) [4], in which a single Minkowski brane is embedded in a five-dimensional anti-de Sitter (AdS) bulk.

Thin branes can, however, only be treated as a rough approximation, because any fundamental underlying theory, be it quantum gravity or string or M-theory, must contain a fundamental length beyond which the classical spacetime description is impossible. It is therefore necessary to justify the infinitely thin brane approximation as a well-defined limit of a smooth structure, a thick brane, obtainable as a solution of coupled gravitational and matter field equations. Such a configuration is then required to be globally regular, stable, and properly concentrated around a three-dimensional surface that is meant to describe the observed spatial dimensions. Topological defects emerging in phase transitions with spontaneous symmetry breaking (SSB) are probably the best candidates for this role.

It should be mentioned that the evolution of the Universe, according to modern views, contained a sequence of phase transitions with SSB. A decisive step toward cosmological applications of the SSB concept was made in 1972 by Kirzhnits [5]. He assumed that, as in the case of solid substances, a symmetry of a field system, existing at sufficiently high temperatures, could be spontaneously broken as the temperature falls down. A necessary consequence of such phase transitions is the appearance of topological defects. The first quantitative analysis of the cosmological consequences of SSB was given by Zel’dovich, Kobzarev, and Okun’ [6]. Later, the SSB phenomenon and various topological defects were widely used in inflationary...
University models and in attempts to explain the origin of the large-scale structure of the Universe (see, e.g., [7, 8]).

The properties of global topological defects are generally described with the aid of a multiplet of scalar fields playing the role of an order parameter. If a defect is to be interpreted as a braneworld, its structure is determined by the self-gravity of the scalar field system and may be described by a set of Einstein and scalar equations.

In this paper, we analyze the gravitational properties of candidate (thick) braneworlds with the four-dimensional Minkowski metric as global topological defects in extra dimensions. Our general formulation covers such particular cases as a brane (domain wall) in five-dimensional spacetime (one extra dimension), a global cosmic string with winding number \( n = 1 \) (two extra dimensions), and global monopoles (three or more extra dimensions). We restrict ourselves to Minkowski branes, because most of the existing problems are clearly seen even in these comparatively simple systems; on the other hand, in the majority of physical situations, the inner curvature of the brane itself is much smaller than the curvature related to brane formation, and, therefore, the main qualitative features of Minkowski branes should survive in curved branes.

Brane worlds as thick domain walls in a five-dimensional bulk have been discussed in many papers (see, e.g., [9] and references therein). Such systems were analyzed in a general form in [10, 11] without specifying the symmetry-breaking potential; it was shown, in particular, that all regular configurations should have an AdS asymptotic form. Therefore, all possible thick branes are merely regularized versions of the RS2 model, with all concomitant difficulties in matter-field confinement. Thus, it has been demonstrated [11] that a test scalar field has a divergent stress–energy tensor infinitely far from the brane, at the AdS horizon. The reason for that is the repulsive gravity of the RS2 and similar models: gravity repels matter from the brane and pushes it towards the AdS horizon. To overcome this difficulty, it is natural to try considering a greater number of extra dimensions. This was one of the reasons for us to consider higher-dimensional branes.

We study the simplest possible realization of this idea, assuming a static, spherically symmetric configuration of the extra dimensions and a thick Minkowski brane as a concentration of the scalar field stress–energy tensor near the center. The possible trapping properties of gravity for test matter are then determined by the behavior of the so-called warp factor (the metric coefficient acting as a gravitational potential) far from the center, and we indeed find classes of regular solutions where gravity is attracting.

Some of our results repeat those obtained in [12, 13], which have discussed global and gauge (‘t Hooft–Polyakov-type) monopoles in extra dimensions; a more detailed comparison is given in Section 7.

The paper is organized as follows. In Section 2, we formulate the problem, introduce spacetimes with global topological defects in the extra dimensions, write the equations and boundary conditions, and demonstrate a connection between the possibility of SSB and the properties of the potential at a regular center. In Section 3, we briefly discuss the trapping problem for RS2-type domain-wall models and show that they always have repulsive gravity and are unable to trap matter in the form of a test scalar field. Section 4 is devoted to a search for regular global monopole solutions in higher dimensions by analyzing their asymptotic properties far from the center. All regular configurations are classified by the behavior of the spherical radius \( r \) and by the properties of the potential. This leads to separation of “weak gravity” and “strong gravity” regimes, related to maximum values of the scalar field magnitude.

In the weak gravity regime, the spherical radius \( r \) tends to infinity along with the distance from the center. Such moderately curved configurations exist without any restrictions of fine-tuning type. If the scalar field magnitude exceeds some critical value, the radius \( r \) either tends to a finite value far from the center or returns to zero at a finite distance from the center, thus forming one more centers, which should also be regular. Some cases require fine tuning of the parameters of the potential, and, hence, one may believe that static configurations can only exist if the scalar and gravitational forces are somewhat mutually balanced.

In Section 5, we show that, in contrast to domain walls, global monopoles in different regimes do provide scalar field trapping on the brane. Section 6 is a brief description of numerical experiments with the Mexican hat potential admitting shifts up and down, equivalent to introducing a bulk cosmological constant. Their results confirm and illustrate the conclusions in Section 4. Section 7 summarizes the results.

2. PROBLEM SETTING

2.1. Geometry

We consider a \((D = d_0 + d_1 + 1)\)-dimensional spacetime with the structure \( M^{d_0} \times \mathbb{R}_x \times S^{d_1} \) and the metric

\[
\begin{align*}
 ds^2 &= e^{2\gamma(u)} \eta_{\mu\nu} dx^\mu dx^\nu \\
 &\quad - (e^{2\alpha(u)} du^2 + e^{2\beta(u)} d\Omega^2).
\end{align*}
\]

Here,

\[
\eta_{\mu\nu} dx^\mu dx^\nu = dt^2 - (dx)^2
\]

is the Minkowski metric in the subspace \( M^{d_0} \),

\[
\eta_{\mu\nu} = \text{diag}(1, -1, \ldots, -1);
\]

\(d\Omega\) is a linear element on a \(d_1\)-dimensional unit sphere \( S^{d_1} \); \( \alpha, \beta, \) and \( \gamma \) are functions of the radial coordinate \( u \).