The interest in considering the processes of the reflection and propagation of sound in waveguides with an elastic layered bottom is related to the development of the methods and means for the diagnostics and reconstruction of the bottom characteristics, as well as to the urgency of mineral, oil, and gas prospecting at a sea shelf by acoustic methods. To model the sound interaction with the ocean bottom, the reflection coefficients of plane waves are conventionally used. The mathematical methods of describing plane wave interactions with elastic layered media have been developed in a number of classical works [1–6]. The matrix method of calculating the plane wave reflection coefficients and refraction indices, which was developed by Molotkov [3, 4], was successfully used in our previous paper [7] for interpreting experimental data. The method of tensor impedances allowing the calculation of the reflection coefficients for media with piecewise-constant elastic and inertial parameters is presented by Machevariani et al. [5]. In this case, the problem is reduced to a set of Riccati differential equations, which can be solved by the Runge–Kutta method. Prikhod’ko and Tyutekin [6] used the impedance method for numerically calculating the elastic wave characteristics in continuously layered solid media. Many publications [8–11] are devoted to studies of the reflection and propagation of sound in layered elastic media. However, the use of complex bottom models in the modeling of sound reflection and propagation began only in recent years. Our interest is in the study of both the necessity to take into account the bottom parameter variation with depth [12, 13] and the relation between the sea bottom reflectivity and its acoustic characteristics [14–17].

In this paper, we numerically realized the Thomson–Haskell matrix method [18, 19], which is conventionally used for describing layered elastic media. According to this method, each elastic medium is characterized by a fourth-order matrix, and the whole system is described by a matrix that is obtained by multiplying the characteristic matrices of all media. The elements of this matrix allow one to calculate the interference reflection coefficients and refraction indices, as well as the dispersion characteristics of the interference waves. However, the domain of the validity of the Thomson–Haskell computational scheme turns out to be basically restricted. In this connection, we passed from the fourth-order characteristic matrices to the sixth-order matrices first suggested by Dunkin [20] and Thrower [21]. Theoretically developed by Molotkov [3, 4] and numerically realized in this paper, the Dunkin–Thrower approach allowed one to increase the accuracy of computer calculations of the reflection coefficients of plane waves. A program based on the matrix method was tested, and the test calculations were compared with data on the angular dependences of reflection losses for the reflection from a hypothetical turbidite layers [22]. The numerical calculations were used to illustrate the behavior of the frequency-angular resonances and absorption effects for various
layered elastic/liquid bottom models. Changes in the structure of resonances (their location, width, and amplitude) with the variations in the radiation frequency, grazing angle, and shear elasticity in the layer and the underlying halfspace was considered. These investigations were performed with the aim of developing the resonance methods for the reconstruction of the parameters of a layered elastic bottom.

The physical model of the medium is presented as a set of \( n \) plane-parallel elastic layers overlaying an elastic halfspace. The \( z \) axis is directed upward, along the normal to the horizontally stratified elastic layers \( j = 1, 2, \ldots, n \), where \( n \) is the number of the elastic layers. Within a sedimentary elastic layer, the density \( \rho_j \), the velocities of the longitudinal \( c_{lj} \) and transverse \( c_{tj} \) waves, and the attenuation coefficients of the longitudinal \( \eta_j \) and transverse \( \eta_t \) waves are deemed to be constant. The water column (\( 0 \)) and the elastic base (\( \infty \)) are homogeneous halfspaces. In all layers, including the elastic halfspace, the attenuation effects are taken into account by introducing the complex velocities of the longitudinal and transverse waves \( c_j = c_{re} + ic_{im} \). In turn, the wave numbers will also be complex quantities.

We consider only vertically polarized waves the \( SV \) type, for which the components of the displacement vector \( \mathbf{U} \) are confined in the \( (x, z) \) plane and the displacement along the \( y \) axis is absent. The displacement vectors can be written in terms of the scalar \( \varphi \) and the vector \( \psi \) potentials

\[
\mathbf{U} = \nabla \varphi + \mathbf{rot} \psi. 
\]

For the \( SV \)-wave, the potential \( \psi \) has a single \( y \)-component in an elastic medium and equals zero in water. In the Cartesian coordinates, the displacements in each \( j \)th layer are expressed in terms of the potentials \( \varphi_j \) and \( \psi_j \):

\[
\begin{align*}
U_x &= \partial \varphi_j / \partial x - \partial \psi_j / \partial z, \\
U_z &= \partial \varphi_j / \partial z + \partial \psi_j / \partial x,
\end{align*}
\]

these potentials satisfying the Helmholtz equations

\[
\begin{align*}
\Delta \varphi_j + \alpha_j^2 \varphi_j &= 0, \\
\Delta \psi_j + \beta_j^2 \psi_j &= 0,
\end{align*}
\]

where \( \alpha_j^2 = k_{ij}^2 - \xi_j^2, \beta_j^2 = k_{ij}^2 - \xi_j^2, \) and \( \xi_j = (\omega/c_{0j}) \sin \theta_j \). The relation of the normal \( \sigma_{n,j} \) and the tangential \( \sigma_{t,j} \) components of the stress tensor to the potentials \( \varphi_j \) and \( \psi_j \) has the form

\[
\begin{align*}
\sigma_{n,j} &= 2\mu_j (\partial^2 \varphi_j / \partial x^2 - \partial^2 \psi_j / \partial z^2), \\
\sigma_{t,j} &= -\lambda_j \partial^2 \varphi_j / \partial x^2 \\
&+ (\lambda_j + 2\mu_j) \partial^2 \varphi_j / \partial z^2 + \partial^2 \psi_j / \partial x \partial z,
\end{align*}
\]

where \( \lambda_j \) and \( \mu_j \) are the Lame constants; \( c_{ij} = \sqrt{(\lambda_j + 2\mu_j)/\rho_j} \) and \( c_{tj} = \sqrt{\mu_j/\rho_j} \). The given layered elastic system is excited by a plane wave of the unit amplitude \( \varphi_0^+ = 1 \), which is assumed to arrive from the liquid halfspace. The wave system in the liquid and elastic halfspaces is written as

\[
\begin{align*}
\varphi_0(z) &= \varphi_0^+ \exp(-i\alpha_0z) + \varphi_0^- \exp(i\alpha_0z), \\
\psi_0(z) &= 0, \\
\varphi_n(z) &= \varphi_n^+ \exp(-i\alpha_nz), \\
\psi_n(z) &= \psi_n^+ \exp(-i\beta_nz),
\end{align*}
\]

where \( \varphi_0 = V \) is the reflection coefficient in the liquid halfspace, and \( \varphi_n^+ = W_j \) and \( \psi_n^+ = W_l \) are the refraction indices of the longitudinal and the transverse waves in the elastic halfspace. The wave amplitudes are

\[
\begin{align*}
\varphi_j &= \varphi_j^+ \exp(i\alpha_jz) + \varphi_j^- \exp(-i\alpha_jz), \\
\psi_j &= \psi_j^+ \exp(i\beta_jz) + \psi_j^- \exp(-i\beta_jz),
\end{align*}
\]

where \( \varphi_j^+ \), \( \varphi_j^- \), \( \psi_j^+ \), and \( \psi_j^- \) are some arbitrary functions that characterize the elastic waves propagating in the positive (with the superscript \(-\)) and negative (with the superscript \(+\)) directions of the \( z \)-axis. Substituting expressions (6) in the boundary conditions [1] and performing the differentiation at the boundaries, we obtain \( 4(n+1) \) equations in \( 4(n+1) \) unknowns. We introduce the column vector \( \mathbf{Z}_j = (\varphi_j^+, \varphi_j^-, \psi_j^+, \psi_j^-)^T \), the diagonal matrix \( L_j = [\exp(i\alpha_j h_j), \exp(-i\alpha_j h_j), \exp(i\beta_j h_j), \exp(-i\beta_j h_j)] \), and the characteristic matrix of the 4th order \( A_j \), for the \( j \)th layer, where \( h_j \) is the layer thickness [3, 4]. For the system of \( n \) elastic layers and the elastic halfspace, the following matrix equation is valid:

\[
\mathbf{Z}_n = \mathbf{D} \times \mathbf{Z}_\infty,
\]

where \( \mathbf{D} = A_{n-1}^{-1} \times A_{n-1} \times A_{n-1} \times \ldots \times A_{j+1} \times A_j \times A_j \times \ldots \times A_1 \times L_1 \times A_1 \times A_1 \times A_1 \times A_0 \) is the matrix of the 4th order with the elements \( D_{lm} \), where \( l, m = 1, 2, 3, 4 \), and this matrix characterizes the elastic halfspace. For the combined description of the liquid and layered elastic halfspaces, with allowance for (6), we can write six boundary conditions. In this case, the liquid halfspace is characterized by the matrix of the 2th order \( Q_{\nu} \).