Fermionic string from Abelian Higgs model with monopoles and Θ-term

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The four-dimensional Abelian Higgs model with monopoles and Θ-term is considered in the limit of large mass of the Higgs boson. We show that for Θ=2π the theory is equivalent, at large distances, to summation over all possible world-sheets of fermionic strings with Dirichlet-type boundary conditions on the string coordinates. © 1996 American Institute of Physics. [S0021-3640(96)00314-3]

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There are processes in quantum field theory in which a particle description is not convenient. Examples are QCD at low energies and some astrophysical phenomena in the early Universe. It is therefore of interest to study string theories which follow from field theories.

For example, the quantum theory of Abrikosov–Nielsen–Olesen (ANO) strings can be obtained from the Abelian Higgs model (AHM). It happens that for thin ANO strings the theory is local. The effective action for thin ANO strings contains a rigidity term with the negative sign. The string theory with the Nambu–Goto and rigidity terms may have problems with unitarity, the presence of a tachyon in the spectrum, and crumpling of the string world-sheet (see, e.g., the review). But such problems are absent for the Neveu–Schwarz–Ramond (NSR) string in ten dimensions. This string is equivalent (at the stable point of the β-function for the rigidity term) to the string theory with action containing the rigidity term plus the topological Wess–Zumino–Novikov–Witten (WZNW) term. In the present paper we give an example how a similar theory can be obtained from the four-dimensional AHM with monopoles and Θ-term in the limit of large mass of the Higgs boson.

We start with the following partition function in Euclidian space–time:

\[
Z = \int \left[ D\tilde{z}\mu \right] D\mu \, D\Phi \exp \left\{ -\int d^4x \left[ \frac{1}{4} (F_{\mu\nu} + \tilde{F}_{\mu\nu}(\tilde{z}))^2 + \frac{1}{2} |D_\mu \Phi|^2 + \lambda (|\Phi|^2 - \xi^2)^2 + i \Theta e^2 \frac{1}{32\pi^2} \epsilon_{\nu\alpha\beta}(F_{\mu\nu} + \tilde{F}_{\mu\nu}(\tilde{z}))(F_{\alpha\beta} + \tilde{F}_{\alpha\beta}(\tilde{z})) \right] \right\},
\]

(1)

where

\[
D_\mu = \partial_\mu - ieA_\mu - ie\tilde{A}_\mu.
\]
\[ \epsilon_{\mu \nu \alpha \beta} \partial_{\mu} \vec{F}_{\alpha \beta}(\vec{z}) = \frac{4\pi}{e} \int_{\mathcal{C}} \frac{d\vec{z}}{e} \delta^{(4)}(x - \vec{z}), \quad \partial_{[\mu} \vec{A}_{\nu]} = \vec{F}_{\mu \nu}, \]  

(2)

and \( j_\mu = (1/4\pi) \epsilon_{\mu \nu \alpha \beta} F_{\alpha \beta} \) is the conserved monopole’s current: \( \partial_{[\mu} j_{\nu]} = 0 \), \( \vec{z}_\mu \) is the position of the monopole; \( \int [\mathcal{D}\vec{z}_\mu] \) is the functional integral over all closed paths, (the measure is well known, see, e.g., Ref. 1); \( \mathcal{C} \) are the trajectories of the monopoles defined by \( \vec{z}_\mu \); \( \Phi = |\Phi| e^{i\theta} \) is the Higgs field with the standard integration measure: \( \mathcal{D}\Phi = \mathcal{D}\text{Re} \Phi \mathcal{D}\text{Im} \Phi = |\Phi| \mathcal{D}\Phi \mathcal{D}\theta \).

The theory (1) can be considered as the low energy limit of the \( SU(2) \) Georgi–Glashow model with the \( \Theta \)-term and with additional breaking of the gauge \( U(1) \) symmetry. This model is known to have ANO strings and ‘t Hooft–Polyakov monopoles as the solutions of the classical equations of motion.\(^2\) At low energy the monopoles can be regarded as Wu–Yang type ambiguities in the gauge potential \( A_\mu \) (Ref. 1). In Eq. (1) we explicitly write these ambiguities as \( \vec{F}_{\mu \nu}(\vec{z}) \).

Since at the center of the ANO strings \( \text{Im} \Phi = \text{Re} \Phi = 0 \), the phase \( \theta \) is singular on two-dimensional surfaces that are world-sheets of ANO strings. The character of the singularity is:

\[ \partial_{[\mu} \partial_{\nu]} \theta(x, \vec{x}) = 2\pi \epsilon_{\mu \nu \alpha \beta} \Sigma_{\alpha \beta}(x, \vec{x}), \]

\[ \Sigma_{\alpha \beta}(x, \vec{x}) = \int_{\Sigma_C} d^2 \sigma e^{a b} \partial_{[a} \vec{x}_{\alpha}] \partial_{[b} \vec{x}_{\beta]} \delta^{(4)} [x - \vec{x}(\sigma)], \]

where \( \Sigma \) and \( \Sigma_C \) are the sets of all closed surfaces and surfaces open on the monopole’s world-lines \( C \).

Using the Bianchi identity \( (\epsilon_{\mu \nu \alpha \beta} \partial_{\nu} F_{\alpha \beta} = 0) \) and conservation of the monopole current \( (\partial_{[\mu} j_{\nu]} = 0) \) we can rewrite the \( \Theta \)-term as \( (\Theta e^{i/2\pi}) j_\mu A_\mu \) (Ref. 10), which is the interaction of the electric charge of the dyon with the gauge field.

In Eq. (1) \( \mathcal{D}\theta \) contains the integration over functions which are singular on two-dimensional manifolds (3), and we subdivide \( \theta \) into a regular \( \theta' \) and a singular \( \theta^s \) part: \( \theta = \theta' + \theta^s \); \( \theta^s \) is defined by Eq. (3). To simplify the calculations we consider the London limit \( (\lambda \gg 1) \), in which case the radial part \( |\Phi| \) of the Higgs field \( \Phi \) is fixed, \( |\Phi| = \zeta \), and \( \mathcal{D}\theta = \mathcal{D}\theta' \mathcal{D}\theta^s \). After the change of variables from \( \theta' \) to \( \vec{x}_\mu \) and integration over \( A_\mu \) and \( \theta^s \) in (1), we get:

\[ Z = \text{const} \int [\mathcal{D}\vec{x}_\mu] [\mathcal{D}\vec{\alpha}] J(\vec{x}) \exp \left\{ - \int d^4 x \int d^3 y \left[ \pi^2 \epsilon^2 \Sigma_{\mu \nu}(x) \mathcal{D}_m^{(4)}(x - y) \Sigma_{\mu \nu}(y) \right. \right. \]

\[ \left. \left. + \left( \frac{\Theta e}{2\pi} \right)^2 + \frac{1}{4\pi^2} j_\mu(x) \mathcal{D}_m^{(4)}(x - y) j_\mu(y) + \frac{\Theta e}{2\pi} j_\mu(x) \right. \right. \]

\[ \left. \left. \times \mathcal{D}_m^{(4)}(x - y) \partial_\nu \epsilon_{\mu \nu \alpha \beta} \Sigma_{\alpha \beta}(y) \right] + i \Theta \mathcal{L}(\Sigma, C) + i \Theta \mathcal{L}(\Sigma_C, C), \right\} \]

(4)

where \( \vec{x}_\mu \) is the position of the string, \( [\mathcal{D}\vec{x}_\mu] \) assumes both integration over all possible positions and summation over all topologies of the string’s world-sheets \( \Sigma \) and \( \Sigma_C \); \( \mathcal{D}_m^{(4)}(x - y) \) is the Green’s function: \( (\Delta + m^2) \mathcal{D}_m^{(4)}(x - y) = \delta^{(4)}(x - y) \), \( m^2 = e^2 \epsilon^2 \) is the