On Nambu monopole dynamics in a SU(2) lattice Higgs model

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It is shown that an SU(2) Higgs model on a lattice is equivalent to the Georgi–Glashow model in the limit of a small coupling constant between the Higgs and gauge fields. It can therefore be concluded that the transition between the confinement and symmetric phases in a 3 + 1 dimensional SU(2) Higgs model at finite temperature is accompanied by condensation of Nambu monopoles. © 1997 American Institute of Physics. [S0021-3640(97)00121-7]

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According to one of the most popular modern hypotheses, the baryon asymmetry of the universe arose in the process of an electroweak phase transition (see, for example, the review by Rubakov and Shaposhinkov1). On account of the smallness of the Weinberg angle \( \theta_W \) and the insignificance of the fermion effects, this transition is largely determined by the properties of the SU(2) Higgs model. The present letter examines the behavior of the magnetic fluctuations, which can play an important role in a temperature-induced phase transition, in the SU(2) Higgs model.

Let us consider the SU(2) lattice Higgs model with scalar field \( \Phi_x \) in the fundamental representation, the action in which is described by the formula

\[
S[U,\Phi] = -\frac{\beta}{2} \sum_P \text{Tr} U_P - \kappa \sum_x \sum_\mu (\Phi^+_x U_{x,\mu} \Phi_{x+\mu} + \text{c.c.}) + V(|\Phi|).
\]  

(1)

Here \( U_P \) represents the ordered product of the edge elements of the gauge field \( U_{x,\mu} \) over the boundaries faces of the plaquette \( P \), and \( V(|\Phi|) \) is the potential on the field \( \Phi \), and \( |\Phi|^2 = \Phi^+ \Phi \).

On account of the triviality of the homotopy group \( \pi_2(SU(2)) \), there are no topologically stable monopolar defects in this theory. However, “embedded”2 monopoles, the so-called “Nambu monopoles,”3 which are not topologically stable defects, do exist in the theory. These objects are described by the composite field

\[
\chi^a_x = \Phi^+_x \sigma^a \Phi_x
\]  

(2)

(\( \sigma^a \) are Pauli matrices), which behaves under gauge transformations as a scalar field in the adjoint representation. A Nambu monopole is a configuration of fields \( U \) and \( \Phi \) such that the field \( U \) and the composite field \( \chi \), expressed in terms of the fundamental field \( \Phi \)
according to Eq. (2), possess the configuration of a 't Hooft–Polyakov monopole in the Georgi–Glashow model with the field $\chi$ in the adjoint representation and with the gauge field $U$.

Since Nambu monopoles are described solely by the gauge field $U$ and the composite field $\chi$, the dynamics of these monopoles is determined completely by the effective action $S_{\text{eff}}$

$$e^{-S_{\text{eff}}[U,\chi]} = \int D\Phi e^{-S[U,\Phi]} \prod_a \prod_x \delta(\chi_a^a - \Phi^+_a \sigma^a \Phi_a).$$

To calculate the action $S_{\text{eff}}$ it is convenient to study the following parametrization of the field $F$:

$$F = e^{i\varphi}\Psi, \quad \Psi = \rho \left( \begin{array}{c} \cos \alpha \ e^{i\theta} \\ \sin \alpha \end{array} \right),$$

where $\varphi, \theta \in [-\pi, \pi)$, $\alpha \in [0, \pi/2]$, and $\rho \in [0, +\infty)$. The fields $\rho, \alpha,$ and $\theta$ can be expressed in terms of the field $\chi^a$ with the aid of Eqs. (2):

$$\theta = \arctan \frac{\chi^2}{\chi^1}, \quad \alpha = \frac{1}{2} \arctan \frac{\sqrt{(\chi^1)^2 + (\chi^2)^2}}{|\chi|}, \quad \rho = |\chi|,$$

whence

$$\Psi = \frac{1}{\sqrt{2 (|\chi| - |\chi^3|)}} \left( \chi^1 + i\chi^2 \right).$$

Using the relation for the modulus of the field $\Phi$, $|\Phi|^2 = |\chi| = \sqrt{\sum_{a=1}^3 (\chi^a)^2}$, and the measure

$$\int_{-\infty}^{+\infty} D\Phi \ldots = \int_{-\pi}^{\pi} D\varphi \prod_{a=x}^{+\infty} \prod_x \frac{1}{|\chi_x|} \ d\chi_x^1 \ d\chi_x^2 \ d\chi_x^3 \ldots,$$

we obtain for the effective action (3)

$$S_{\text{eff}}[U,\chi] = -\frac{\beta}{2} \sum_p \text{Tr} \ U_p + S[\chi] + V(|\chi|),$$

where the new potential on the field $\chi$ is determined by the expression

$$V(|\chi|) = V(\sqrt{|\chi|}) + \sum_x \ln|\chi_x|,$$

and the interaction of the fields $U$ and $\chi$ is

$$e^{-S[\chi]} = \int_{-\pi}^{\pi} D\varphi \left( \kappa \sum_x \sum_\mu R_{x,\mu} \cos(\varphi_{x,\mu} - \varphi_x + A_{x,\mu}) \right).$$

In this formula we introduced the notation

$$\Psi^+_x U_{x,\mu} \Psi_{x,\mu} = R_{x,\mu} e^{iA_{x,\mu}}.$$