Vortex drag in the quantum Hall effect

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A new model of momentum and electric field transfer between two adjacent 2D electron systems in the quantum Hall effect is proposed. The drag effect is due to momentum transfer from the vortex system of one layer to the vortex system of another layer. The remarkable result of this approach is a periodic change of sign of the dragged electric field as a function of the difference between the layer filling factors.

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The drag effect in double-layer two-dimensional electron systems has been the subject of intense recent interest, especially at high magnetic fields, where the quantum Hall effect (QHE) exists.1 Theoretical explanations of the observed drag effect mostly reduce to the ordinary Coulomb interaction between electrons occupying the Landau levels in the two layers.2 These explanations are not much related to the quantum Hall effect. In the present letter a new mechanism for the momentum transfer between the layers is proposed, which exists exclusively under QHE conditions. For simplicity we shall consider integer filling factors.

The model is based on a treatment of the quantum Hall liquid as a superfluid state of Chern–Simons charged bosons.3,4 The ground state $\phi = \phi_1 + i \phi_2$ has a quasilong-range phase correlation5 and is uniform in the mean field approximation at ‘‘magic’’ filling factors $f_i = n_i$, where $i = 1, 2, \ldots$ is an integer.

In accordance with this point of view on the QHE, vortex ($H>H_0^i$) or antivortex ($H<H_0^i$) excitations are created away from the filling factor $f_i$, with concentrations $N^v_i = |H-H_0^i|/\Phi_0$, where $H$ is the external magnetic field, and $H_0^i$ is the magnetic field corresponding to the filling factor $f_i$. The magnetic flux carried by the vortex (antivortex) is $\Phi_0 = -\Phi_0$, where $\Phi_0 = hc/e$ is the flux quantum. If the ground state carries a supercurrent $j^\text{ext}$ the vortices experience an average force $F^\text{ext}$, which corresponds to the force acting on the vortices in an ordinary superconductor. Formally the force is a result of current–current interactions between the external supercurrent $j^\text{ext}$ and the vortex supercurrent $j^v$ (Ref. 6). The Hamiltonian of the interaction is $H_{\text{int}} = \Lambda (j^v \cdot j^\text{ext})$, where $\Lambda$ is an interaction constant. The negative of the derivative of the Hamiltonian $H_{\text{int}}$ with respect to the vortex position is the external force $F^\text{ext}$. The gauge invariant expression for the supercurrent of charged bosons is the same as for an ordinary superconductor, but the charge of the Cooper pair $2e$ needs to be changed to the boson charge $q$. The charged boson supercurrent is $j = -(\Lambda c)^{-1}(A-hc/2pq\nabla \chi)$, where $\chi$ is the phase of the boson.
ground state \( \phi \). Assuming that the function \( \phi \) is single-valued, we find the external force \( F^{\text{ext}} \) to be

\[
F^{\text{ext}} = \frac{\pm \Phi_0}{q_i c} [j^{\text{ext}} \times \mathbf{e}_z],
\]

where \( c \) is the speed of light, \( \mathbf{e}_z \) is the direction of the magnetic field \( \mathbf{H} \), and the sign \( + \) \((-\) \) corresponds to the vortex (antivortex). The value \( q_i \) is the boson charge in units of the electron charge: \( q = q_i e \). A similar expression for the force in an external electric field \( E \) has been found in a different approach: \( F^{\text{ext}} = \frac{e^2}{\hbar c} \Phi_0 E \).

Using the relation between the current and the electric field in the QHE:

\[
j^{\text{ext}} = \sigma_{xy} E = q_i e^2 / \hbar E,
\]

we obtain \( F^{\text{ext}} = F^{\text{ext}}_2 \).

There are several possibilities for the vortex motion in the external current \( j^{\text{ext}} \). A simple picture is considered here. At the temperature \( T = 0 \) K all the vortices are pinned by the disorder, and the current \( j^{\text{ext}} \) flows without any dissipation. There is no momentum transfer to the vortex system at \( T = 0 \) K. At a finite temperature \( T > 0 \) K, the vortices can jump from one point to another as a result of thermal fluctuations. The external current \( j^{\text{ext}} \) induces an average momentum (creep) of the vortices in the direction of the force \( F^{\text{ext}} \). Thus, at the temperature \( T > 0 \) K there is momentum transfer from the external current to the vortex system. The average nonzero momentum of the vortices in the first layer will relax and be partially converted into momentum of the vortices of the second layer. Actually, the most effective channel of momentum transfer between the layers is not clear. A suitable candidate is the phonon system.

In this paper the momentum transfer between the layers is studied phenomenologically. Let us consider the momentum transfer between the vortices at some QHE resistance minimum of the first layer \( (H^0_i) \) and the vortices at some QHE resistance minimum of the second layer \( (H^0_2) \) (see Fig. 1). The average momentum of the vortices in the first (second) layer is \( \mathbf{P}_1 \) (\( \mathbf{P}_2 \)). The total force exerted on the vortex system by the external current \( j^{\text{ext}} \) (the layers are squares of unit area) is \( N^i v \mathbf{F}^{\text{ext}}_i (1) \), where \( i = 1, 2 \) is the layer index now.

Newton’s equations for the momenta of the vortices are \( (H - H^0_i = N^i v \Phi_0) \):

\[
d\mathbf{P}_1 / dt = 1/(q_1 c) [j^{\text{ext}}_1 \times (\mathbf{H} - \mathbf{H}^0_1)] - \mathbf{P}_1 / \tau_1 - \mathbf{F}^{\text{int}},
\]

\[
d\mathbf{P}_2 / dt = 1/(q_2 c) [j^{\text{ext}}_2 \times (\mathbf{H} - \mathbf{H}^0_2)] - \mathbf{P}_2 / \tau_2 + \mathbf{F}^{\text{int}},
\]

where the \( \tau_i \) are the momentum relaxation rates and \( \mathbf{F}^{\text{int}} \) is the interlayer drag force.

In the experiment of Ref. 1 an additional electric field \( E^{\text{ext}}_2 \) was applied to the second layer to cancel the current \( j^{\text{ext}}_2 \). In this case Eq. (3) is transformed into

\[
d\mathbf{P}_2 / dt = - \mathbf{P}_2 / \tau_2 + \mathbf{F}^{\text{int}}.
\]

For small perturbations of the vortex distribution function the interlayer drag force is proportional to the vortex momentum \( \mathbf{P}_1 \). At a small value of vortex concentration \( N^1 v \), the total drag force is proportional to the concentration \( N^2 v \). Therefore the interlayer drag force \( \mathbf{F}^{\text{int}} \) can be approximated by