Cross Sections and Asymmetries for Elastic $\pi^3\text{He}$ Scattering in the Energy Region around the $\Delta$ Resonance*

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Abstract—For a polarized target, $\pi^3\text{He}$ interaction is studied in the fixed-center approximation with all rescatterings included. Only the $P_{33}$ wave is taken for the $\pi N$ interaction. The nuclear wave function is taken either as a sum of Gaussian functions or as a Faddeev wave function in the $s$-wave approximation. The differential cross sections and asymmetries for elastic $\pi^3\text{He}$ scattering at the laboratory energies of $T_p = 142$, 180, and 256 MeV are calculated. The results are compared with experimental data.

1. INTRODUCTION

Investigation of pion scattering off light nuclei requires its reliable theoretical description. An exact relativistic treatment of systems with four or more particles is beyond the present computational possibilities, so that simplifications are unavoidable. Current studies of $\pi$–trinucleon reactions are mostly performed on the basis of the optical potential model [1], which treats the nucleus as a single particle interacting with the pion via some effective potential. This may be a good approach at low energies when the pion wavelength $\lambda_\pi$ is larger than the internucleon distance $R$. However, at energies above 100 MeV, when $\lambda_\pi \leq R$, the optical potential model does not seem natural, and alternative approaches deserve attention. A viable alternative seems to be the fixed-scattering-center model, which emerges in the limit $m_p/m_N \rightarrow 0$. In this model, all intermediate nuclear states are taken into account, although the variation of their energies is neglected. If one retains only the ground state in the sum over intermediate nuclear states in the fixed-center model, then the optical-model results are recovered (without the contribution of the nucleus to the energy denominators). Thus, the fixed-center model represents an improvement of the optical potential model in that it simplifies (i.e., it is simple and essentially reduces to a separable form, which simplifies the calculations substantially). On the other hand, the pion energy still remains considerably smaller than the nucleon energy. Therefore, one can hope that neglecting the nucleon recoil might be a reasonable approximation.

In [3], we introduced a model that is based on the fixed-center approximation for elastic $\pi^3\text{H}$ and $\pi^3\text{He}$ interactions and which takes completely into account multiple pion rescattering. A simple Gaussian wave function for the nuclear ground-state was used. The present paper reports on the calculations with the Faddeev ground-state wave functions and also on the application of the model to polarized targets.

2. GENERAL FORMALISM

In the fixed-center approximation, the determination of the amplitude of the interaction with a nucleus reduces to evaluating the sum of the diagrams shown in Fig. 1.

The exact treatment of these graphs requires the introduction of 216 amplitudes. In the present calculations, the spin-tensor interaction in the elementary block of Fig. 2 was replaced by an averaged one. As a result, the number of independent amplitudes was reduced to 27.

The basic Faddeev-like amplitudes are $M_{ik}$, where $i$ and $k$ refer to the number of initial and final nucleons ($i, k = 1, 2, 3$). Each $M_{ik}$ is a $3 \times 3$ matrix in both spin and isospin. The final system of linear equations for the amplitudes $M_{ik}$ has the form

$$M_{ik} = R_1 P^{(i)} \delta_{ik} + R_1 \sum_{l \neq i} P^{(i)} W_{li} M_{lk}, \quad (1)$$

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Here, \( \frac{d^3}{d^3} r_1d^3 r_2(\exp(i(k- k') \cdot r_1))F_{(T,S)}(r_{ik}) \),
\[
\mathcal{A}_2^{(T)} = \frac{3\lambda^2}{\sqrt{2k_02k_0'S}} \sum_{S = 1/2, 3/2}(k'P^{(S)}k)
\]
\[
\times \int d^3 r_1d^3 r_2\exp(i(k \cdot r_1 - k' \cdot r_2))G_{(T,S)}(r_{ik})
\]
with
\[
F_{(T,S)}(r_{ik}) = \Psi_A^*(r_{ik})M_{11(T,S)}(r_{ik})\Psi_A(r_{ik}),
\]
\[
G_{(T,S)}(r_{ik}) = 2\Psi_A^*(r_{ik})M_{21(T,S)}(r_{ik})\Psi_A(r_{ik}).
\]

The formula for the amplitude \( \mathcal{A} \) can be represented in a form similar to that for the case of \( \pi N \) interaction,
\[
\mathcal{A}^{(T)} = a^{(T)}(k' \cdot k) + ib^{(T)}(\sigma \cdot k' \times k),
\]
\[
a^{(T)} = \frac{1}{3}(\mathcal{A}_{1,1}^{(3)} + \mathcal{A}_{2,1}^{(3)} + 2(\mathcal{A}_{1,3}^{(3)} + \mathcal{A}_{2,3}^{(3)})),
\]
\[
b^{(T)} = \frac{1}{3}(\mathcal{A}_{1,1}^{(3)} + \mathcal{A}_{2,1}^{(3)} - (\mathcal{A}_{1,3}^{(3)} + \mathcal{A}_{2,3}^{(3)})).
\]

The amplitudes \( \mathcal{A}_{1,5}^{(T)} \), which results from integration with respect to the angles in (4), are determined by triple integrals of the form

\[
\mathcal{A}_{1,5}^{(T)} = \frac{24\pi^2\lambda^2}{\kappa}\int_0^{\infty} r_1dr_1\sin(\kappa r_1)
\]
\[
\times \int_0^{+1} r_2^2dr_2 \int dz F_{(T,S)}(r_1, r_2, z),
\]
\[
\mathcal{A}_{2,5}^{(T)} = \frac{12\pi^2\lambda^2}{\sqrt{2k_02k_0'}}\int_0^{+1} r_2^2dr_2
\]
\[
\times \int dz_2 G_{(T,S)}(r_1, r_2, z_2)D(kr_1, \kappa r_2, \cos \delta, z_2).\]

In these expressions, the variables \( z \) and \( z_2 \) are \( \cos(r_{13}, r_{23}) \) and \( \cos(r_{13}, r_{23}) \), respectively, and the notation \( \cos \delta = -\frac{1 - \cos \theta_\pi}{2} \) and \( \kappa = -2k\cos \delta \), where \( \theta_\pi \) is the c.m. angle of the pion, is used. The function \( D \),