DETERMINATE SYSTEMS

A Complete-Order Hybrid Identifier for Multiprogrammed Stabilization

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Abstract—Stabilization of a family of programmed motions of a linear control system with a first-order nonlinear hybrid state identifier is studied. Such an identifier and a multiprogrammed hybrid control are designed in the form of nonlinear feedback. A theorem on sufficient conditions for the existence of a solution to the formulated problem is proved.

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1. INTRODUCTION. MULTIPROGRAMMED STABILIZATION

Representation of the right sides of a system of differential equations having a predefined finite set of solutions and design of controls for realizing a given family of programmed motions and guaranteeing their stabilization in the sense of asymptotic Lyapunov-stability were first studied in [1, 2]. Special attention is paid in [2] to the representation of such controls for linear stationary control systems. In these papers, the results are illustrated with examples on the control of a mechanical system described by second-kind Lagrange equations and control of motion of charged particles in an electromagnetic field.

Let us consider the linear control system

\[ \dot{x} = Ax + Bu + f(t), \]  

where \( x \in \mathbb{R}^n \) is the phase state vector, \( u \in \mathbb{R}^r \) is the control vector, \( A \) and \( B \) are constant real matrices of suitable size, and \( f(t) \) is a real continuous vector function defined for \( t \in (-\infty, +\infty) \).

**Problem 1** [2] (multiprogrammed stabilization). Design a control

\[ u = u(x, t) \]  

(2)

for realizing given programmed motions \( x_j = x_j(t) \) under open-loop controls \( u_j = u_j(t), j = \overline{1, N} \) such that the programmed motions \( x_j(t) \) under control (2) are asymptotically Lyapunov-stable.

**Note 1.** Here we shall not study the design of open-loop controls \( u_j(t) \) and the corresponding motions \( x_j(t) \) since they are known. For the sake of definiteness, we assume that the open-loop control \( u_j(t) \) and programmed motion \( x_j(t) \) are determined from the problem of transfer of system (1) from a given initial state to a given final state. Therefore, for a closed-loop system (1) with an open-loop control \( u_j(t) \), there exists a particular solution \( x_j(t) \) under chosen initial and final data. In other words, every pair of \( u_j(t) \) and \( x_j(t) \) satisfies a t-identity on the interval on which these functions are defined

\[ \dot{x}_j(t) = Ax_j(t) + Bu_j(t) + f(t), \quad j = \overline{1, N}. \]
Theorem 1 ([2]). Let the following conditions hold:

1. the system \( \dot{x} = Ax + Bu \) for \( u = Cx \) has an arbitrarily large stability margin found from a proper choice of the matrix \( C \), and

2. programmed motions \( x_j(t) \) are distinguishable for \( t \geq t_0 \geq 0 \), i.e., \( \inf_{t \geq 0} \|x_i - x_j\| > 0 \), \( i \neq j \); then there exists a control \( (4) \) realizing the programmed motions \( x_j(t) \) such that every motion is asymptotically Lyapunov-stable.

Note that the number \( N \) of programmed motions \( x_j(t) \) is not related to the dimensions of system (1) and control space. Theorem 1 is proved in [3]. Control (2) can be expressed as

\[
u(x, t) = \sum_{j=1}^{N} \left( u_j + C(x - x_j) - 2u_j \sum_{i=1, i \neq j}^{N} \frac{(x_j - x_i)(x - x_j)}{(x_j - x_i)^2} \right) p_j(x, t), \tag{3}\]

where

\[
p_j(x, t) = \prod_{i=1, i \neq j}^{N} \frac{(x - x_i)^2}{(x_j - x_i)^2}, \quad j = 1, N. \tag{4}\]

In fact, this is a Lagrange–Silvester interpolation polynomial, in which programmed motions \( x_j(t) \) are nodes and open-loop controls \( u_j(t) \) are values of \( u_j \). By construction, functions (3), (4) are such that \( p_j(x_j(t), t) \equiv 1 \), \( p_j(x_i, t) \equiv 0 \), \( i \neq j \) and \( u(x_j(t), t) = u_j(t), j = 1, N \). An expression of the type \( (x_j - x_i)(x - x_j) \) or \( (x_j - x_i)^2 \) denotes the scalar product of the respective vectors. In formulas (3) and (4) and in what follows, where there is no room for confusion, we shall not indicate the argument \( t \) in vector functions \( x_i, x_j, u_j, \ldots \). Since the time-dependence of the functions \( p_j(x, t) \) is implicit and expressed through \( x_i(t) \) and \( x_j(t) \), we use the notation \( p_j(x) \) in the left side of (4).

The closed-loop system (1) with control (3), (4) is a multiprogrammed automaton capable of realizing any programmed motion \( x_j(t) \) from a given family, depending on the choice of initial data and ensuring the asymptotic stability of the motion. In [4, 5], problem 1 is studied for linear and bilinear systems with incomplete feedback. Algorithms for designing complete-order continuous identifiers are designed in [4, 5] and Luenberger identifiers in [6], in which the deviation vectors \( x - x_j \) in (3) can be replaced by their estimates \( \hat{x} - x_j \).

2. FORMULATION OF THE PROBLEM

For realizing control (3), (4) in an applied problem, we must have a continuous flow of information on the deviation vectors \( y_j(t) = x(t) - x_j(t) \) for all programmed motions \( x_j(t) \). But such a question does not arise for a discrete controller.

Note that problem 1 was solved under the assumption that the vectors of deviation \( x - x_j \) from programmed modes were measurable. We now change the condition. Let us assume that the measuring device is defined by the equation

\[
z_j(t) = Ry_j(t), \tag{5}\]

where \( z_j(t) \in R^m \) is a measurement vector and \( R \) is a constant real \((m \times n)\) matrix. Equation (5) is also known as the output equation [7] and the vector \( z_j(t) \) is called the output of the system. Knowing the output \( z_j(t) \), we must find an estimate \( \hat{y}_j(t) \) of the vector \( y_j(t) \) such that [7]

\[
y_j(t) - \hat{y}_j(t) \to 0, \quad t \to +\infty. \tag{6}\]

If such an estimate exists, it can be used to design a control similar to control (3), (4).