A Frequency Adaptive Control for Multidimensional Systems

A. G. Aleksandrov* and Yu. F. Orlov**

*Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia
**Moscow State University, Moscow, Russia

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Abstract—An adaptive control is designed for a multidimensional system with unknown constant coefficients under bounded polyharmonic disturbances containing an infinite number of harmonics of unknown amplitudes and frequencies. It uses a very small test signal. The control aim is to ensure given bounds for the forced oscillations in the output of the system and controller. Adaptation is based on finite-frequency identification of the system and a closed-loop system. By way of example, an adaptive control of a real physical system is given.

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1. INTRODUCTION

There are several trends in adaptive control under unknown bounded external disturbances. The first of them is related to reference-model systems. Adaptive controls for such systems were first designed without regard for external disturbances [1–3]. Subsequently these systems, as shown in [4], were found to lose stability under the action of external disturbances. Thus there emerged a large number of papers concerned with the design of adaptive control algorithms for stabilization in these and other tracking systems under external disturbances. Typical results of this trend are outlined in [5, 6]. This approach can be illustrated with the example of [5], where an $LQ$-optimization problem is solved for a system with unknown coefficients [5]. To solve a problem expressed as Riccati equations, the true coefficients of the system are replaced by their quasi-estimates found by the gradient method. They may considerably differ from true coefficients since the identification problem has no solution under unknown external disturbances (if the test signals described below are neglected). Therefore, quasi-estimates are possible values for the coefficients consistent with the input and output of the system. Adaptation process is shown to converge to some unknown tracking error. Other adaptation methods without the use of quasi-estimates are described in [7].

The method of recurrent aim inequalities [8, 9] laid the foundation for the second trend. In this trend, adaptive control aim is expressed as constraints (margins) for the deviation of the steady output of the system. The solution of the $l_1$-optimization problem [10, 11] is extended in [12, 13] to a system with unknown coefficients. In these papers, quasi-estimates are determined by a special gradient method for the deviation of the steady-state output to be minimal. It is not easy to realize the adaptive control algorithm numerically. This is the cost to be paid for the best adjustment accuracy it provides under unknown coefficients and arbitrary bounded external disturbances.

Therefore, many papers restrict external disturbances to a narrower class. An unknown constant disturbance is used in [14]. The adaptive control algorithm for this case is simple in realization. In [15], external disturbance is defined by a piecewise-constant bounded function with a known
frequency range. Adaption aim is a given characteristic polynomial of a closed-loop system (which is also used in [16, 17], where the external disturbance is an arbitrary bounded function). Coefficients of the system are estimated with an adaptive observer, and the control law (which is formed from these estimates and state vector estimate) contains a test signal.

In frequency adaptive control [18], as in the second trend, control aim is the magnitude of the steady-state output of a system. External disturbance is the sum of an infinite number of harmonics of unknown amplitudes and frequencies and sum of amplitudes bounded by a known number. The system and a closed-loop system are identified by the finite-frequency identification method [19], in which a system or a closed-loop system is excited by a test signal—sum of harmonics, whose number is not greater than the dimension of the state space of the system or the closed-loop system. The frequency of the test signal must not be the same as that of the external disturbance. This condition somewhat narrows the class of external disturbances, and is verified during identification.

In the adaptation methods described above, the controller is continuously adjusted, whereas controller parameters are adjusted after large time intervals (adaptation intervals) in frequency adaptive control for guaranteeing the linearity of the model of the system on these intervals (while in other methods, the model is nonlinear and conditions cannot be easily found such that the input and output do not take unduly large values during adaptation). Therefore, the adaptation algorithm can be numerically realized without any serious difficulties [20].

In this paper, results of [18] are extended to multidimensional systems. Here we encounter two difficulties. This first is the determination of a relation between the steady values of adjusted variables and weight coefficients of the $H_{\infty}$-norm of the transfer matrix of the closed-loop system. The second is the determination of adaptation termination conditions. Termination is obviously implemented by comparing the matrices describing the system at the current and preceding adaptation intervals. For this purpose, these matrices must be uniquely represented. Such a comparison is possible only if matrices are represented in canonical form, which in this paper is taken to be the Luenberger column observable canonical form [22].

In Section 2, we formulate the adaptive control design problem and solve it in Section 3 for a system with known coefficients. Sections 4 and 5 are devoted to identification of a system (independently and in a closed-loop system, respectively). In Section 6, we state conditions for the adaptation process to converge. In Section 7, we describe an adaptive control for a gyroplatform.

2. FORMULATION OF THE PROBLEM

Let us consider the linear stationary system described by the equations

$$\begin{align*}
\dot{x}_p &= A_p x_p + B_p (u + f), \quad y = z = C_p x_p, \quad t \geq t_0, \\
\dot{x}_c &= A_c x_c + B_c y, \quad u = C_c x_c,
\end{align*}$$

(1)

where $x_p(t) \in \mathbb{R}^n$ is the state vector of system (1), $x_c(t) \in \mathbb{R}^n$ is the state vector of controller (2), $u(t) \in \mathbb{R}^m$ is the control vector, $y(t) \in \mathbb{R}^r$ is a vector of measured variables, $z(t) \in \mathbb{R}^r$ is a vector of adjusted variables, $f(t) \in \mathbb{R}^m$ is a vector of unmeasurable external disturbances—bounded polyharmonic functions

$$f_j(t) = \sum_{k=1}^{\infty} f_{jk} \sin \left( \omega_k^j t + \varphi_{jk} \right), \quad j = 1, m,$$

(3)

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1 This paper is a revised version of report [21] read at the 15th IFAC Congress in Barcelona.