DETERMINATE SYSTEMS

Probabilistic Characteristics of Paramacrosystems

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Abstract—Consideration is given to macrosystems called paramacrosystems with different elements, with stochastic behavior, and states of finite capacity. Thus is fulfilled the interval between two classical capacity characteristics of states: Fermi, with one element, and Einstein, with unlimited number of elements in each state. Using the apparatus of generating functions, we obtained expressions for probabilistic characteristics (functions of probability distribution and “physical” entropies) for four classes of paramacrosystems with corresponding combinations of distinguishable and indistinguishable elements and states with finite capacities.

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1. INTRODUCTION

To study a great deal of system phenomena and processes in nature and society, a two-level representation is useful: the level of elements and the level of the system as a whole. The utility of this representation is connected with difficulties and sometimes with impossibility to investigate the whole system as it is complicated and we do not have enough of knowledge about nature and its performance mechanisms. On the other hand, elements (elementary particles) of the system are more prime by definition and we can always choose such elements the properties of which can be identified.

Apparently, this approach was formulated for the first time in physics where the terms “microlevel,” “macrolevel,” and “macrosystem” appeared [1]. Then it came to other scientific disciplines; gradually it became a tool for theoretic and experimental research; and the theory of macrosystems is based on it.

One of its branches is connected with the study of equilibria in macrosystems with stochastic behavior of indistinguishable elements that can occupy states of Fermi-, Einstein-, and Boltzmann-types (one element, any number of elements in one state, or small “in the mean” occupancy of states) [2]. The indicated phenomenology of states is interpreted as Fermi, Einstein, and Boltzmann statistics.

However, there always has been a question about the arrangement of statistics of macrosystems in which states can be occupied by \(2, 3, \ldots, l < \infty\) elements. These statistics occupy the interval between Fermi- and Einstein-statistics and are called parastatistics [3]. An important characteristic of parastatistics is its order \(l\), i.e., the limited number of elements that may occupy the state. In terms of order, the Fermi-statistics is “parastatistics of order 1;” the Einstein-characteristics is “parastatistics of order \(\infty\).” We shall identify macrosystems in which the mechanism of microstates formation is characterized by parastatistics of order \(l\) as paramacrosystems of order \(l\).

Determination of equilibrium macrostates remains burning for paramacrosystems. This problem is not only cognitive but also pragmatic since a lot of applied problems related to urban and regional

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planning, demo-economic prediction, routing in computer networks, and etc., have “parastatistical”
properties.

Identification and research of equilibria consists of two stages: definition of probabilistic char-
acteristics of macrostates and definition of a realized (equilibrium) macrostate. The first stage is
based on the notion of microstates and related a priori information. The second stage is based on
the classical variational principle distinguishing a macrostate in the admissible set of macrosystems
such that it maximizes (minimizes) probability, entropy, information, or something else adopted in
place of the characteristic of the set of admissible macrostates.

The paper is devoted to the definition of probabilistic characteristics of macrostates in para-
macrostates with distinguishable and indistinguishable elements and states on condition that mi-
crostates are equally possible in them.

2. MICRODESCRIPTION AND CLASSIFICATION OF MACROSYSTEMS

Let us consider a macrosystem consisting of \( Y \) elements that may occupy states in the set \( S \) of
states. The set \( S \) is a union of disjoint subsets \( S_1, \ldots, S_m \) with \( G_1, \ldots, G_m \) states; each of them
has capacity \( l_1, \ldots, l_m \). If all \( l_i = 1 \), we have a macrosystem with Fermi-states. If all \( l_i = \infty \), we
have a macrosystem with Einstein-states.

The macrostate is described by a vector \( N = \{ N_1, \ldots, N_m \} \) where \( N_i \) is a number of elements
(the occupation number) occupying the subset \( S_i \). We can see from the definition of macrostate
that the elements and states are faceless in it. On the other hand, its characteristics are made of
some unions of microstates which are defined by a complete microscopic description (of elements
and states).

The completeness of the microdescription is connected with the adopted classification procedure.
We assume that there are two functionals; one of them is a generalized characteristic of the element;
the other is a generalized characteristic of the state. If this functional takes on similar values for all
elements, then the elements are indistinguishable (of course, in terms of the selected functional). If
these values are strictly different, then the elements are distinguishable. The similar classification
occurs for states.

Thereby, we obtain four classes of macrosystems:

- \( DD \) is a macrosystem (elements and states are distinguishable);
- \( ID \) is a macrosystem (elements are indistinguishable and states are distinguishable);
- \( DI \) is a macrosystem (elements are distinguishable and states are indistinguishable);
- \( II \) is a macrosystem (elements and states are indistinguishable).

The given classification can be illustrated by the following examples with a random behavior
of elements. Let us consider a distribution of specialists over vacancies in the labor market. Let
specialists have different qualifications and work experience; the vacancies differ in the level of
requirements and salary. In this case, we have a model of \( DD \)-macrosystem.

Let us imagine that there is a subgroup of specialists within the mentioned group; they have
the same qualification and work experience, i.e., they are indistinguishable. At the same time, the
vacancies are different. This situation is quite typical of the Russian market as the information
about the qualification and work experience is misrepresented; thus it is convenient for the employer
to consider all seekers equal. In this case, we have a model of \( ID \)-macrosystem.

A reversed situation occurs when specialists of different level have to distribute over equal
vacancies, which corresponds to the model of \( DI \)-macrosystem.

Finally, there may be a situation when specialists of equal level distribute over identical vacan-
cies, i.e., according to the model of \( II \)-macrosystem.