CONTROL IN THE PRESENCE OF UNCERTAINTY

Application of the Set-Theoretic Approach to Accounting of Uncertainties in the Solution of Vector Optimization Problems

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Abstract—The problem of optimization of a complex system by the vector criterion is considered in terms of the set theory. The suggested approach to its solution is based on the formal description of a set of the weight factors of partial indices of effectiveness by the system of inequalities proceeding from the nonstrict and incomplete system of preferences of the decision-maker (DM), which enables us to reduce the calculation of values of both the integral and the guaranteeing criterion of optimality to the solution of a simple geometric problem.

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1. INTRODUCTION

In optimization of the structure and parameters of a control system of a complex technical object (in particular, a flying vehicle), it is necessary to ensure a correct accounting of the multiple character of its problems and conditions of functioning [1]. Here, the basic problem is the calculation of the estimate of effectiveness of the control system by the aggregate of heterogeneous and contradictory, as a rule, partial (local) indices (for example, the indices of quality of a transient process: the times of actuation, readjustment, variability, etc.).

This problem relates to the problems of vector optimization. However, the conventional methods of optimization by the vector criterion are either too cumbersome and laborious (as methods on the basis of expert estimates) or do not make it possible, in the formalization of a set of indices for the organization of operative optimization procedures, to reflect completely and comprehensively special features of this set (for example, the lexicographic approach) [2].

Hence, the need arises in the mathematical model of the estimate of effectiveness in the set of indices that would combine to the maximum degree the completeness and correctness of the description of this set with the possibility of developing the optimization algorithms that are effective in the computational aspect.

In this article, an attempt is made to construct such a model.

2. PROBLEM OF ACCOUNTING FOR UNCERTAINTY OF THE EFFECTIVENESS ESTIMATE BY THE AGGREGATE OF INDICES

We will consider, in the general form, the problem of optimization by the aggregate of indices of the control system as a certain design solution.
APPLICATION OF THE SET-THEORETIC APPROACH

Let the design solution \( y \in Y \), where the \( Y \) is an admissible set of these solutions, be estimated by the aggregate from the \( S \) scalar indices \( f_i(y), i = 1, S \). It is necessary to select in the \( Y \) the best version \( \tilde{y} \).

To this statement of the problem there corresponds a model of vector optimization

\[
\tilde{F}(\tilde{y}) = \text{opt } \tilde{F}(y), \quad \tilde{F} = (f_1, \ldots, f_S),
\]

where \( \text{opt} \) is the optimization operator of the effectiveness vector, which reflects the system of preferences of the DM.

Here, the key problem is the problem of accounting of the ambiguity of the operator \( \text{opt} \) and, hence, the optimal solution \( \tilde{y} \). To remove this ambiguity, it is necessary to state the rule of calculation of the effectiveness estimate by the aggregate of indices, which in the mathematical sense is the functional of components \( f_1, \ldots, f_S \) of the effectiveness vector \( \tilde{F} \)

\[
F(y) = \varphi[f_1(y), \ldots, f_S(y)].
\]

In the scalar optimization model

\[
\tilde{y} = \arg\min_{y \in Y} \sum_{i=1}^{S} a_i f_i(y)
\]

the functional (2) has the form of the linear convolution \( F(y) = \sum_{i=1}^{S} a_i f_i(y) \), where the \( a_1, \ldots, a_S \) are the weight factors of local criteria, which model the system of preferences of the DM and satisfy the conditions

\[
\sum_{i=1}^{S} a_i = 1, \quad a_i > 0, \quad i = 1, S.
\]

The geometric interpretation of the factor \( a_i \) is shown in Fig. 1.

In minimization of the effectiveness estimate, the best solution corresponds to the point of tangency of the hyperplane \( \sum_{i=1}^{S} a_i f_i(y) \) and the surface \( F_0 \). At the \( S = 2 \), this is the point of tangency of the straight line \( a_1 f_1 + a_2 f_2 \) and the curve \( F_0 \) (Fig. 1).

Thus, the assignment of the convolution and, hence, the decision problem reduce to the choice of values \( a_i \in [0 \ldots 1] \), which is conventionally unique both in the scalar optimization model, where these values are preset in the explicit form, and in the vector model, where the convolution \( F \) is prescribed by the strict ranking of indices [2].