Control of Wheel System under Uncertainty

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Abstract—Consideration was given to the nonholonomic mechanical systems with rolling or the wheel systems such as mobile robot, car, or wheeled tractor. Analysis was confined to the kinematic models with regard for the dynamics of the controlling drives. Control of system motion along a given trajectory (planar smooth curve) was studied. The designed control law stabilizes this motion in large in the basic variables. The main result lies in solution of the problem of control under uncertainty when only sufficiently general dynamic characteristics of the wheel system drive are known.

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1. INTRODUCTION

The kinematic models of the nonholonomic mechanical systems with rolling describe rather well the motions of physical wheel systems. That is why they currently find wide use in the control problems among which one may cite the theoretical problems of stability of autonomous carriers, problems of motion planning, controllability, and so on [1–10]. These models are used also to solve many purely applied problems of control such as car stabilization on the road, automatic parking, robot motion in an environment with obstacles, and so on [8, 11–15].

The present paper solves the problem of stabilization of motion of a wheel system (WS) along the given trajectory. Problems of this kind are frequently encountered. For example, it may be required to maintain WS motion along a given trajectory with the aim of performing certain construction or other technological operations (cabling, trenching, and so on). In agriculture, the wheeled tractors perform independently a wide range of operations such as ploughing, planting, additional fertilizing, weeding, and so on [12, 13]).

It deserves noting that the aforementioned trajectory problem of WS control where it is required to stabilize a set of points in the WS state space ([3], V.I. Zubov, Vysshaya Shkola, 1973) is an important, though special, one. Solution of the general problem of control (point stabilization, for example) offers certain difficulties. In the general case, the smooth stationary laws of control do not enable one to stabilize motions of the nonholonomic system [2]. Smoothness of the control law is an important condition because one has to allow for the dynamics of the system drives. Exponential stability is required to compensate for various disturbances.

The problem of WS control is solved under natural assumptions. The trajectory is defined as a planar smooth curve. Only one control, that of the WS front axle, is used in the system. It is assumed that the wheel system moves along the trajectory with a certain speed set up by the driver using the gas pedal and gear box and it is required to maintain only the “transversal stability.”

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The main problems are related to the need to stabilize WS motion in large in the basic variables and to allowance for the incompletely defined front axle drive dynamics.

The WS dynamics is described in Section 2. Section 3 presents the problem of control and formulates the tasks of the present study. The problem of WS control is reduced to a constructive form in Section 4 and solved in a simplified form in Section 5 where the discontinuous laws of control are admissible. The smooth law is constructed in Section 6, and Section 7 discusses constructions of the laws of control enabling one to reach the aims of WS control with higher precision.

2. WHEEL SYSTEM AS A CONTROL PLANT

The wheel system under consideration is diagrammed in general in Fig. 1. The WS has body and driving rear axle, as well as the controllable front axle. The state of the body is characterized by the angle $a$ and the coordinates $x$ and $y$ of some its point $p$ having speed of magnitude $v$. The state of the front axle is characterized by the controlled angle $b$.

With regard for the above notation, the WS motion obeys the equation system

$$\dot{x} = v \cos a, \quad \dot{y} = v \sin a, \quad \dot{a} = v \tan b/L, \quad \dot{b} = f(b, u, t),$$

(2.1)

where the first three equations describe the translational and angular WS motions, and the last equation describes the motions of the drive of the controlled front axle, $u$ is control, and $v, L = \text{const} > 0$. If the system point $p$ lies on the given curve $S$, then the aim of WS control obeys $p \in S$.

The first two equations of (2.1) describe the mechanical WS relations (Fig. 1) and reflect the assumption that the rear wheels do not slip in the direction of the front wheels. A similar assumption about the front wheels allows one to construct the third equation of the system. These equations represent a kinematic model of the mechanical wheel system. Models of this kind are under careful examination [1]. For example, some studies additionally allow for the inertial characteristics of the mechanical wheel system, which enables one to study the impact of the external forces on the WS [8, 11]. We also note that the considered system with rolling represents a classical subject of study of the analytical mechanics of nonholonomic systems [16–23].

The problems of WS control usually disregarded the drive dynamics [1], and in system (2.1) the variable $b$ was considered instead, for example, as some control parameter from the class of continuous or smooth time functions [5]. The present paper allows for the drive dynamics in the last equation of system (2.1).

We note that the problem of WS control was actually disregarded within the framework of the well-known urgent problem of control under uncertainty. Indeed, this problem usually was studied for the case of certain, completely defined WS motion Eqs. (2.1) [1, 5], whereas in the physical WS’s it is usually the hydraulic drives that are used for the front axles [24]. The description of the WS drive dynamics usually involves uncertain parameters. Consequently, it is only natural to assume