Generalized Clos Networks

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Abstract—Proposed was an extension to the class of multistage Clos networks based on the interstage connection circuit described by the balanced incomplete block designs considered in the combinatorics.

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1. INTRODUCTION

The basic \( R \)-port Clos network is shown in Fig. 1. It has three respective full \( r \times m \), \( r \times r \), and \( m \times r \) stages, is completely defined by the parameters \( m \), \( n \), \( r \), and reconfigurable for \( r = m \) or nonblocking for \( m \geq 2r - 1 \).

The \( t = 2k + 1 \)-stage Clos network is constructed recursively on the assumption that the second stage consists of \( m \) Clos networks with \( 2k - 1 \) stages. In this case, the reconfigurable multistage Clos network has \( R_t = r^{k+1} \) ports. The reconfigurable network has individual conflictless schedules for any permutation of data between the input and output ports. Its complexity is equal to \( S_t = tR_t^{(k+2)/(k+1)} \) switching points. The nonblocking network allows one to route a new conflictless path between any pair of free input and output ports without modifying the existing paths. Its complexity is \( S_t < K_tR_t^{(k+2)/(k+1)} \) for \( K_t \leq (t - 1)^2 \) [1–3].

In comparison with the complexity of the complete \( R_t \)-port switch, the relative complexity of the Clos network is defined as \( S_t/R_t^2 \). The multistage Clos networks are distinguished for relative complexity decreasing with increase in the number of stages upon retention of the switching characteristics such as recognizability or nonblocking.

The complete uniform bipartite graph \( D(X,Y,m,r) \), where \( X \) defines the set of switches of the first and third stages and the vertex set \( Y \) defines the set of switches of the second stage, models the interconnections in the basic Clos network. All vertices of the set \( X \) have the same degree \( m \), and their number is \( |X| = r \). All vertices of the set \( Y \) have the same degree \( r \), and their number is \( |Y| = m \). The parameters of the graph \( D(X,Y,m,r) \) are related by the equality \( m|X| = r|Y| \). Graph \( D(X,Y,5,3) \) is exemplified in Fig. 2.

The main idea of the present paper lies in replacing the complete uniform bipartite graph \( D(X,Y,m,r) \) by a quasicomplete uniform bipartite graph \( D(X,Y,N,M,m,r,\sigma) \) where the parameters \( X \), \( Y \), \( m \), and \( r \) are defined as before, but the numbers of vertices of each set are changed (\( |X| = N \geq r \), \( |Y| = M \geq m \)) and the constant number \( \sigma \) of paths of length two between any two vertices of the set \( X \) is defined. In the quasicomplete bipartite graph such paths pass through different vertices of the set \( Y \) and their number \( \sigma \) is the same for any vertices from \( X \). An example of the graph \( D(X,Y,3,6,4,2,2) \) is depicted in Fig. 3. We note that for \( \sigma = m \) the graph \( D(X,Y,N,M,m,r,\sigma) \) becomes \( D(X,Y,m,r) \).

The aim of replacing the interconnection graph in the basic model lies in increasing the number of vertices of the set \( X \) without modifying the degree of vertices of the sets \( X \) and \( Y \) which leads
to an increase in the number of ports of the multistage network or a decrease in the complexity at comparable number of ports. Replacement of the basic model leads to construction of a new class of generalized multistage Clos networks including the classical multistage Clos networks as special cases. It is required to determine for this class the conditions for retention of the properties of switching and reduction of the relative network complexity with increased number of stages.

The aforementioned quasicomplete uniform bipartite graphs have been studied for a long time in the combinatorics as the balanced incomplete block designs $B(N, M, m, r, \sigma)$ [4]. The present paper continues the studies [5–9] on using the block diagrams to define and describe the system communication networks.

2. BLOCK DIAGRAMS AND BIPARTITE GRAPHS

**Definition 1.** By the balanced block diagram $B(N, M, m, r, \sigma)$ is meant an assemblage of $M$ blocks comprising $N$ different elements so that each block has precisely $r$ different elements included each precisely in $m$ blocks and each pair of elements is included precisely in $\sigma$ blocks [4].

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**Fig. 1.** Basic three-stage Clos network.

**Fig. 2.** Full bipartite graph $D(X, Y, m, r)$ with the parameters $m = 5$ and $|X| = 3$, $r = 3$ and $|Y| = 5$.

**Fig. 3.** Quasicomplete bipartite graph $D(X, Y, N, M, m, r, \sigma)$ with the parameters $m = 4$ and $|X| = N = 3$, $r = 2$ and $|Y| = M = 6$, $\sigma = 2$. 