On the Calendar Planning Problem with Renewable Resource

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Abstract—We consider a strongly NP-hard calendar planning problem with constraints on resource consumption and job ordering. One characteristic feature of our problem setting is that resource consumption intensities by different jobs may change during their processing, and resource availability depends on time. To solve the problem, we construct an integer programming model and develop a dynamic programming algorithm. We distinguish a special case of the problem that can be solved in pseudopolynomial time. We show numerical experiments on randomly generated test examples.

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1. INTRODUCTION

In production control, it is very important to develop algorithms to solve calendar planning problems with constraints on resource consumption and the order of fulfilling the jobs. The most widely studied calendar planning problem deals with renewable resources and overall job processing time criterion [1–3]. Renewable resources [1, 4] are characterized by the fact that each job has its own intensity of resource consumption at each specific moment of time. The total intensity of resource consumption over all jobs being processed at time moment $t$ should not exceed the amount of resource available at that time.

Note that in [1–3], functions defining resource constraints (resource consumption intensities for each job and functions that determine the amount of available resources) are assumed to be constant. In this work, we consider the calendar planning problem with renewable resources in a more general form, when functions defining resource constraints are assumed to be piecewise constant. Problems of this kind arise, for instance, in chemical production [5], when raw material received through a pipeline is distributed between reactors that are processing it. The undistributed part of the stream is not stored and does not participate in further production, so at every time moment, raw material consumption cannot exceed the intensity of the stream. The stream intensity depends on the time, and intensities of raw material consumption by various jobs change over the course of their processing.

In this work, we construct an integer linear programming model for this problem and develop an exact algorithm for constructing the optimal job schedule by using an idea from [1]. We specify a special case of the problem in which this algorithm turns out to be polynomial.

The paper is organized as follows. In Section 2, we give the problem setting and formulate the integer linear programming model. In Section 3, we propose a dynamical programming algorithm to solve this problem. In Section 4, we show results of numerical experiments. The last section discusses our main results.
2. PROBLEM SETTING

Consider $m$ machines. For each machine $l$, a sequence of jobs $I_l = (i^l_1, \ldots, i^m_l)$ that have to be processed on this machine in the said order is given. Here $u_l$ denotes the number of jobs for machine $l$, $l = 1, \ldots, m$. We denote the set of all jobs by $I$. Interrelations between the jobs on one or different machines are given by relations of the form $i \rightarrow j$, where job $j$ cannot begin before job $i$ is over. This structure can be represented by a directed acyclic graph $G = (I, E)$, where $I$ is the set of vertices and $E = \{(i, j) : i, j \in I, \ i \rightarrow j\}$ is the set of arcs.

The jobs use one kind of renewable resource. Each job $i \in I$ is characterized by its duration $p_i \in \mathbb{Z}^+$ (here and in what follows $\mathbb{Z}^+$ denotes the set of positive integers) and resource consumption intensity specified as follows. The total length of a job $i \in I$ is divided into $a_i$ time intervals (periods), during each of which the resource consumption intensity for this job is presumed constant; $d_{ik} \in \mathbb{Z}^+$ is the duration of a period with index $k$ of a job $i$, and $r_{ik} \in \mathbb{R}_+$ is the resource consumption intensity for the job $i$ during period $k$, $i \in I$, $k = 1, \ldots, a_i$ (here and in what follows $\mathbb{R}_+$ denotes the set of nonnegative real numbers).

In different time moments over the planning horizon, which lasts for $H \in \mathbb{Z}^+$, the amount of available resource may be different. Suppose that there are $b_{\max}$ periods during each of which the amount of available resource is constant. $T^*_b \in \mathbb{Z}^+$ is the time when period $b$ begins, and $R_b \in \mathbb{R}_+$ is the amount of resource available at every moment of period $b$, $b = 1, \ldots, b_{\max}$ (here and in what follows $\mathbb{Z}^+$ denotes the set of nonnegative integers). Note that $T^*_1 = 0$, the time when $T^*_b$ begins coincides with the time when period $b - 1$ ends, $b = 2, \ldots, b_{\max}$, and the last period ends at time $H$.

For every time moment $t$, $0 < t \leq H$, the total resource consumption intensity over all jobs being processed at time moment $t$ should not exceed the amount of resource available at the current moment. We cannot put jobs on hold, and all jobs must finish before time moment $H$. The latter condition is not critical for $H$ sufficiently large.

Let $S_i$ denote the time when job $i$ begins, $i \in I$. We need to construct such a schedule $S = \{S_i\}$ for the jobs, taking into account technological procedure $E$ and resource constraints, that the total time $C_{\max}$ when all jobs are done is minimized.

Note that the jobs of a single machine form a simple chain in the graph $G$ because they must be processed in a given order. However, this restriction does not narrow down the calendar planning problem (see, e.g., [1, 4]) in which the jobs are not bound to machines and there is an arbitrary partial ordering linking the jobs. Indeed, such a set of jobs can be divided into nonintersecting subsets each of which is a simple chain in polynomial time [1]. Then, introducing a separate machine for each of these chains, we get the problem in our setting.

Statement. The calendar planning problem with renewable resource is strongly NP-hard even for an empty partial ordering.

For this problem, we construct an integer linear programming model. To simplify the exposition, we denote by $A_i = \{1, \ldots, a_i\}$ the set of indices of the periods of job $i$ during each of which the resource consumption intensity by this job is constant, $i \in I$.

Since the durations of the job periods are integer-valued, and the amount of available resource changes only at integer-valued moments of time, it suffices to look for the optimal solution of this problem among the schedules in which each job begins and ends at integer-valued moments of time.

Let us define the problem variables:

- $y_{ik}$ is a binary variable that determines whether period with index $k \in A_i$ of a job $i \in I$ begins at time moment $t - 1$ (in this case, $y_{ik}^t = 1$) or not ($y_{ik}^t = 0$), $t = 1, \ldots, H$;
- $C_{\max}$ is the total time it takes to complete all jobs.