On Segmenting Logistical Zones for Servicing Continuously Developed Consumers

A. L. Kazakov, A. A. Lempert, and D. S. Bukharov

Institute of System Dynamics and Control Theory, Siberian Branch, Russian Academy of Sciences, Irkutsk, Russia

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Abstract—We study the optimal placement problem for several logistical objects. A characteristic feature of this problem is the need for sequential segmentation into servicing zones and accounting for population distributed continuously across the entire region. We reduce this problem to a variational calculus problem in a special form. To study this problem, we develop numerical algorithms that are able to determine the optimal placement of a logistical object inside a given segment. The algorithms are based on constructing wavefronts for a light wave emitted from the boundary of the chosen region. The wave moves inside the region, which lets us account for all inhabitants in this region. In constructing the solution, we have accounted for the loss of smoothness in the wavefront, have developed a software implementation for the computational algorithms, and have conducted a numerical experiment for a number of model problems.

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1. INTRODUCTION

Optimization problems arising in mathematical modeling in transportation and transport logistics have certain characteristic features related to the non-uniformity and geographical placements of the objects of study; as a result, in certain cases the problems cannot be efficiently solved with classical methods. This leads to the need to develop a methodology for their numerical solution in cases when the construction of an exact solution becomes problematic.

In this work, we study the optimal placement problem for several logistical objects with simultaneous segmentation of logistical zones for consumers located continuously along the entire servicing area. To study this problem, we develop a modification of the optic-geometric approach based on constructing light wave fronts in case when the excitation originates from a certain manifold (as opposed to a point source considered in [1]).

As a rule, discrete methods can be successfully applied to this class of problems. However, sometimes a number of complications arise, e.g., it may become impossible to completely account for specific conditions of the problems (landscape, population distribution, natural or artificial barriers). One way to overcome these difficulties, in our opinion, is to consider these problems in their continuous setting and apply the optic-geometric approach to study them.

Previously, in the works of a correspondence member of the RAS V.N. Ushakov and his students [2, 3] a similar approach has been applied to control problems for moving objects on a plane under phase constraints. Numerical implementation was based on constructing reachability sets. In [4], the optical analogy was used to solve security problems each of which was reduced to a routing problem. The same analogy was also used in [5, 6], where the distribution of a wave was considered as an excitation process on a given manifold of neurons.

In solving the problem of locating logistical objects from the security viewpoint (e.g., rescue points, warehouses for toxic and radioactive waste) one needs to account for all localities in the
given area, including settlements with small population; the consumers are continuously distributed along the entire servicing area, i.e., the population density is given by a continuous function. A key characteristic feature of this problem is the need to sequentially segment the area and determine the optimal placement of a logistical center inside each segment.

In solving placement problems for logistical objects, one needs to organize means of communication (roads, power lines, and information channels) that should be placed with minimal possible costs. This leads to solving the Steiner problem [7], where one needs to connect \( m \) cities with roads (or communication channels) with minimal total length. To get roads of minimal total length, one can introduce branching points (also called Steiner points). This problem was first posed by Fermat for \( m = 3 \) [7, 8] and solved by Torricelli. The general case \((m > 3)\) was studied by the geometer Steiner. This problem has received the most attention in its setting on graphs [9, 10], and a wide array of methods have been developed for it. In [11], it is noted that the tree structure in Steiner problems on a plane and on a graph are the same, so solving the Steiner problem on a graph is assumed to be the same as solving it on a plane.

In this work, we present a mathematical setting and a numerical algorithm for solving the placement problem for several logistical objects with sequential segmentation into servicing zones. We study the Steiner problem of a special form and present an algorithm for solving it. Our results are an immediate development of the work [1].

2. METHOD OF SOLVING THE PROBLEM

In solving the placement problem for logistical objects with continuously distributed consumers, there arises a need to construct wavefronts inside the area in question whose boundary forms the initial manifold of light excitation sources. We describe a modification of the optic-geometric approach used to construct wavefronts in optically nonuniform media.

An optical medium is characterized by a set of points with a certain permeability coefficient \( f(x,y) \) that changes the speed of light passing through it. The greater the permeability of an optical medium, the greater will be the speed of light.

By the Huygens principle, every point of the medium reached by a light wavefront becomes an independent source of light. We represent the optical medium as a rectangular system of coordinates \( x \) and \( y \). Suppose that in a certain bounded region \( D \) there exists a basis curve along which the light excitation occurs at time moment \( t = 0 \). From every point of the basis curve we go for a certain distance \( \Delta s \) perpendicular to the minimal time interval \( \Delta t = \varepsilon \) and thus construct a curve close to the basis which is known as the equidistant line [12]. This line forms the manifold of secondary light sources. A similar construction can be performed until the entire region \( D \) is filled up. As a result, every point of the region \( D \) will have a light wavefront going through it.

The distance \( \Delta s \) characterizes a certain straight line segment of the path. Since each segment is placed perpendicular to the basis curve, we know the direction of our construction that ensures a correct identification of the connections between neighboring points. This feature lets us uniquely determine the path of our movement.

Let us show that the resulting path is actually minimal with respect to time. Suppose that the optical medium is nonuniform, \( v(x,y) \) and \( f(x,y) \) are the speed of light and the permeability coefficient at point \((x,y)\) respectively. The distance taken from the initial manifold of light sources equals \( \Delta s = v\Delta t \) \((\Delta t = \varepsilon)\). Since the medium is nonuniform, the speed of light changes from point to point: \( v(x,y) = cf(x,y) \), where \( c \) is the speed of light in vacuum.

If \( f(x,y) \) is given at every point of the region \( D \), an infinitesimal distance \( \Delta s = cf(x,y)\Delta t \) can be taken from all of the basis curve, and then the process can be repeated. If there is an impassable barrier \( D_b \in D \) in the optical medium \( D \) with permeability \( f_b = 0 \), it is impossible to construct a wavefront in region \( D_b \).