Game-Theoretic Model of Agents’ Interaction on a Two-Stage Market with a Random Factor

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Abstract—This paper constructs and examines a game-theoretic model of a two-stage market with arbitrageurs. Arbitrageurs are risk-neutral and operate in the conditions of perfect competition. A random factor affects the outcome on the spot market, making the spot price a random value. We determine the optimal strategies for consumers, producers, and arbitrageurs. In addition, we analyze how the market power of producers depends on the parameters of the model. The results demonstrate that introduction of the forward market substantially reduces the market power of producers.

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1. INTRODUCTION

Heterogeneous product markets (e.g., electricity supply) face the problem of market power restriction for large producers. Due to the reliability of electricity supply, standard antimonopoly regulation methods such as splitting the power supply sector of a national economy into small companies may appear unreasonable. A possible reduction mechanism for the market power of producers lies in the forward market. The paper [3] was among pioneering works that studied the influence exerted by the forward market on the level of market competition. Notably, the authors considered a symmetrical duopoly as the market structure. Two producers successively compete on N forward markets and then on the spot market. By assumption, arbitrage takes no place, i.e., prices are the same during all stages. The results showed that introduction of the forward markets intensifies the competition among producers, as well as increases social welfare. As the number of stages in forward trading tends to infinity, the outcome converges to a competitive market equilibrium.

In the context of our research, the most important result belongs to J. Bushnell, see [5]. The investigator considered a symmetrical oligopoly with n firms described by linear marginal costs. Trading runs in two stages, and the forward price coincides with the spot price. In the special case of constant marginal costs, it appears that the feasibility of forward contracting reduces the market power of producers just like the n-times’ increase in the number of producers operating on the market. However, Bushnell’s hypothesis that the prices on the forward and spot markets are the same disagrees with the real price dynamics of the electricity market [4]. A common situation on real electricity markets is when the spot price turns out somewhat smaller than the forward price. Sometimes the price dynamics has jumps so that the spot price appreciably exceeds the forward price. In addition, the Bushnell model proceeds from the assumption that, on the forward market, customers with high reserve prices gain priority in access to the product. Real markets do not meet this condition.

The paper [7] found a mixed correlated strategy equilibrium provided that customers with reserve prices exceeding the market price purchase the product on the forward market equiprobably. However, the behavior of customers under proportional allocation is irrational taking into account their attitude towards risk. In the current paper, we focus on the model of a two-stage market
with a random price on the spot market. The market includes risk-neutral arbitrageurs whose competition makes the forward price equal to the mathematical expectation of the spot price. Customers also operate in the conditions of perfect competition, choosing freely between the spot market and the forward market. We describe the strategic model of agents’ interaction, characterize the equilibrium behavior of customers and arbitrageurs, as well as evaluate the optimal strategies of producers corresponding to a subgame perfect equilibrium (SPE), also known as an absolute equilibrium. And finally, we estimate the reduction in the market power of producers owing to forward contracting, as well as the probability of a low-price outcome on the spot market.

2. THE MODEL OF INTERACTION AMONG PRODUCERS, ARBITRAGEURS AND CUSTOMERS ON A TWO-STAGE MARKET

As the market structure, consider a symmetrical oligopoly with constant marginal costs $c$. Imagine that $n$ producers (industrial firms) present on the market. A small customer purchases a unit of the product. Customer $b$ is characterized by the reserve price $r_b$. The demand function $D(p)$ is defined by the distribution density $\rho(r)$ of the customers with respect to reserve prices, i.e.,

$$D(p) = \int_{p}^{p_{\text{max}}} \rho(r) \, dr.$$ 

In addition to producers and customers, the market comprises risk-neutral arbitrageurs. They either sell contracts on the forward market and then purchase the product on the spot market, or perform the inverse operation. By assumption, arbitrageurs operate in the conditions of perfect competition.

Let us describe the scheme of interaction among all agents. Trading runs in two stages, first on the forward market and then on the spot market. On the forward market, firms supply their sales volumes $q_a^f$, $a \in \{1, n\}$. Denote by $q^f = (q_a^f, a \in \{1, n\})$ the vector of supplied volumes and by $q^f$ the volume supplied by all producers on the forward market: $q^f = \sum_{a=1}^{n} q_a^f$. Besides producers, the product can be supplied by arbitrageurs. If the latter first sell contracts on the forward market and then purchase the product on the spot market to fulfill their contractual obligations, we designate by $q_{arb}$ the volume supplied by arbitrageurs on the forward market, $q_{arb} > 0$. In the case when arbitrageurs first purchase contracts on the forward market and then sell the product on the spot market, the quantity $|q_{arb}|$ shows the volume purchased by them on the forward market, $q_{arb} < 0$. The notation $q^f_i$ will indicate the volume of products purchased by customers on the forward market.

Each customer $b$ decides to participate in trading on the forward market. Submitting its request, the customer specifies the reserve price $r_b$ and purchases the product if the market price does not exceed the reserve price. Denote by $D^f(p)$ the demand function of customers on the forward market. Actually, this function determines the number of customers deciding to purchase the product on the forward market and whose reserve price is higher than $p$. The price on the forward market $p^f$ follows from the condition $D^f(p^f) = q^f_i = q^f + q_{arb}$.

The spot market operates as the Cournot auction with the residual demand function $D^s(p)$ of customers; this function is evaluated depending on the strategies of all agents. Having purchased the product on the forward market, customers do not participate in trading on the spot market. Therefore, $D^s(p) = D(p) - q^f_i$ for $p < p^f$, and $D^s(p) = D(p) - D^f(p)$ for $p \geq p^f$. Producers supply the sales volumes $q_a^s$, $a \in \{1, n\}$ on the spot market. The spot price $p^s$ balancing the demand and supply meets the condition $D^s(p^s) + q_{arb} = \sum_{a=1}^{n} q_a^s$.

Random events occur between trading on the forward market and trading on the spot market. Let a random factor possess values $i \in \{1, k\}$ with probabilities $w_i > 0$, $\sum_{i=1}^{k} w_i = 1$. Firms can choose the value of supplies on the spot market depending on this random factor. The strategy of a firm is defined by the set $(q_a^f, q_a^s(i); i \in \{1, k\})$ that specifies the volumes of supplies on the forward and spot markets depending on the realization of the random factor. In this case, the price on the spot market represents a random variable whose value $p_i$ can be found from the expression