Effect of the Curvature of the Burning Surface of Condensed Energetic Materials on the Burning Rate

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Combustion of homogeneous condensed energetic materials (CEMs) with a curved burning surface is considered within the framework of the phenomenological theory of unsteady combustion. A dependence of the burning rate on the burning surface curvature is found. It is demonstrated that there exists a limiting surface curvature value above which self-sustained combustion is impossible. This limiting curvature depends on thermophysical and ballistic characteristics of CEMs. The existence of the limiting curvature of the burning surface offers an explanation of the critical conditions of combustion of homogeneous CEMs. Based on this hypothesis, the critical diameters of combustion of several homogeneous CEMs are calculated. The calculated results are in good agreement with available experimental data.

Key words: condensed energetic materials, burning rate, burning surface curvature, critical diameter.

DOI: 10.1134/S0010508211060104

INTRODUCTION

In the theory of combustion of homogeneous condensed energetic materials (CEMs), such as powders, high explosives, active binders, etc., the burning surface is usually assumed to be flat during the entire process. This assumption allows researchers to use one-dimensional models for explaining the laws of combustion (both steady and unsteady) of such substances [1–3].

At the same time, experimental data show that the burning surface is not flat even in homogeneous CEMs. Thus, the cellular-spot structure of the combustion wave of homogeneous CEMs was discovered and studied in [4–7]; it was demonstrated in those publications that the burning surface is covered by inhomogeneities, which can play a key role, especially near the flammability limits.

By an example of the nitroglycerin-ballistite powder (powder NB), it was found [5] that combustion ceased if the inhomogeneity on the surface had a large curvature. To quantify this phenomenon, Marshakov and Istratov [5] used the Michelson–Markstein criterion $\text{Mi} = \frac{R\alpha}{u}$, where $u$ is the burning rate, $R$ is the curvature radius of inhomogeneities on the burning surface, and $\alpha$ is the thermal diffusivity of the condensed phase (c-phase) of the CEMs. Combustion of the inhomogeneity on the powder surface ceased at $\text{Mi} < \text{Mi}_{cr} = 10^{-13}$ [5].

Marshakov and Istratov [5] presented the burning surface curvature profiles for quenched NB powder samples of different diameters. It is seen from these profiles that the burning surface curvature increases with decreasing sample diameter, and the curvature radius becomes approximately equal to the sample radius for the sample with the critical diameter. Zenin et al. [8] noted earlier than the burning surface curvature radius for the nitrocellulose powder (powder N) for the samples with the critical diameter is equal to the sample diameter; nevertheless, more detailed examination of powder N [9] showed that the curvature radius for the sample with the critical diameter is equal to the sample radius, which agrees with the data for powder NB [5].

The CEM burning surface curvature is even more important in the vicinity of walls or substrates, which
can be, among other things, various fillers in composite systems where homogeneous CEMs are used as a binder. Marshakov et al. [10] studied the CEM burning surface shape near walls made of various inert materials and demonstrated that combustion ceased at the same critical values of the Michelson–Markstein criterion as in [5].

All these facts suggest that combustion of CEMs with a curved surface should be essentially different from combustion with a flat surface, while the surface curvature can play a key role in formation of the limiting conditions of CEM combustion.

It is known from the theory of combustion of premixed gas mixtures that the velocity of normal propagation of the flame front depends on its curvature and is expressed by the known Markstein formula [11]

\[ u = u^0 \left( 1 + \alpha \frac{\varphi}{u^0 R} \right), \]

where \( u^0 \) is the velocity of propagation of the plane flame and \( \alpha \approx 1 \) is a dimensionless constant. This effect plays an important role in stabilization of the gas flame front and in formation of its cellular structure [11].

Taking into account the general analogy between the reaction zones in the c-phase of a homogeneous CEM and in premixed gas mixtures [12], we can expect that the burning rate of the CEM with a curved surface will also differ from the burning rate of this CEM with a flat surface, and this difference will be described (at least, formally) by the same Markstein correction [11] proportional to the burning surface curvature.

The present work was aimed at studying the dependence of the burning rate of homogeneous CEMs on the burning surface curvature and analyzing the consequences following from this dependence.

**LAWS OF COMBUSTION OF CEMS WITH A CURVED BURNING SURFACE**

For a particular kinetics of decomposition of a homogeneous CEM, the burning rate as a function of the burning surface curvature can be obtained by solving an appropriate thermokinetic problem with a curved reaction front [13]. Obviously, the proportionality coefficient (Markstein constant) depends appreciably on the chosen kinetic model. As any kinetic scheme is always far from the real pattern of CEM burning because the physical and chemical processes in the condensed and gas phases near the burning surface have many stages and are extremely complicated, such an approach allows only qualitative (not quantitative) dependences to be obtained. In particular, it is possible to derive the Markstein formula with a certain coefficient that cannot be calculated directly, but has to be determined on the basis of experimental data on combustion of CEMs with a curved surface. Obviously, obtaining such data involves severe difficulties, which makes this approach purely formal and unsuitable for practical calculations.

At the same time, we will show below that this dependence can be obtained within the framework of the theory of combustion of homogeneous CEMs in a rather general form, which implicitly takes into account the entire set of physical and chemical processes of CEM combustion. Such a possibility is provided by the phenomenological theory of unsteady combustion [2, 3], which offers a general method for calculating the instantaneous burning rate of CEMs.

In accordance with the phenomenological theory of unsteady combustion [2, 3], the burning rate is a single-valued function of the pressure \( p \) and temperature gradient \( \varphi \) in the c-phase near the burning surface:

\[ u = u(p, \varphi). \] (1)

As this function is identical for steady and unsteady combustion, it can be found from the experimentally determined dependence of the steady burning rate on pressure

\[ u^0 = u^0(p, T_0) \] (2)

and from the theoretical dependence

\[ \varphi = \frac{u^0}{\bar{u}} (T_s - T_0), \] (3)

obtained for the zone of inert heating of the c-phase during steady motion of the flat burning surface. Here, \( T_0 \) is the initial temperature of the charge and \( T_s \) is the temperature of the CEM burning surface, which is also assumed in the phenomenological model [2, 3] to be a single-valued function of the pressure and temperature gradient near the burning surface:

\[ T_s = T_s(p, \varphi). \] (4)

Formally, Eq. (1) can be derived from Eqs. (2)–(4) by solving the equation

\[ u = u^0(p, T_s(p, \varphi) - \omega \varphi / u). \] (5)

As dependence (3) was derived for a flat burning surface, then dependence (1) found by solving Eq. (5) also refers to combustion (steady or unsteady) of a CEM with a flat surface.

It is assumed in the theory [2, 3] that the gas phase is free from inertia and is always tuned to the current burning conditions (pressure, temperature of the burning surface, and temperature gradient in the c-phase near the burning surface).

If the surface becomes curved during CEM combustion for some reasons, the temperature gradient \( \varphi \) differs from the corresponding value for CEM combustion