1. FORMULATION OF THE PROBLEM

Let us consider the attitude motion of a satellite in a central Newtonian gravitational field. The satellite is modeled by a solid body whose linear dimensions are small in comparison with those of the orbit of its center of mass. The last assumption allows one to consider the problem in a restricted formulation: the satellite motion with respect to its center of mass is assumed to have no effect on the motion of the center of mass itself. Further on, it is assumed that the orbit of the center of mass is circular.

A specific case of attitude motion (with respect to the center of mass) is planar motion, at which one of the central principal axes of inertia of a satellite is perpendicular to the plane of its orbit [1]. In a circular orbit the satellite’s planar motions are described by the equation of a mathematical pendulum, and they represent either periodic motions (oscillations and rotations) or the motions asymptotically approaching the unstable position of relative equilibrium. Since the periods of planar oscillations and rotations depend on the initial conditions, the planar periodic motions are unstable in Lyapunov sense against perturbations of coordinates and velocities. Therefore, it makes sense to consider the problem of orbital stability of these motions. In the linear approximation the problem of orbital stability is reduced to the analysis of stability of planar periodic motions of a satellite with respect to spatial perturbations. In such a formulation it was investigated in [2–6].

The aim of this paper is to study orbital stability of the planar oscillations of a satellite whose mass geometry corresponds to a plate, i.e., the equality \( C = A + B \) is valid, where \( A \), \( B \), and \( C \) are the satellite’s principal central moments of inertia. It is assumed that in an unperturbed motion either middle or the largest axis of the ellipsoid of inertia is perpendicular to the orbit plane of the center of mass (the plane of the plate is perpendicular to the orbit plane). The problem of stability is solved in the strictly nonlinear formulation.

The mass geometry approximately satisfying the equality \( C = A + B \) can be typical for modern nano-satellites which have as a power supply the solar arrays whose dimensions substantially exceed the dimensions of the satellite itself.

2. HAMILTONIAN OF PERTURBED MOTION

Let \( OXYZ \) be the orbital coordinate system. We direct \( OX \) axis along the radius vector of the center of mass, axes \( OY \) and \( OZ \) being directed along the vector of the center of mass and along the normal to the orbit plane, respectively. The axes of satellite-fixed coordinate system \( Oxyz \) are directed along the principal central axes of inertia of the satellite. Orientation of the satellite-fixed coordinate system relative to the orbital system is specified by the Eulerian angles \( \psi \), \( \dot{\vartheta} \), and \( \phi \).
The equations of motion of a satellite with respect to its center of mass can be written in the canonical form with the Hamiltonian function

\[ \mathcal{H} = \frac{1}{2} \left[ \frac{1}{2} \frac{\sin^2 \varphi + \theta_A \cos^2 \varphi}{\cos^2 \varphi + \theta_A \sin^2 \varphi} p_\varphi^2 + \frac{1}{2 \theta_A} \cos^2 \varphi \frac{p_\varphi^2}{2 \theta_A} + \frac{(\sin^2 \varphi + \theta_A \cos^2 \varphi)(p_\varphi^2 - 2 p_\varphi p_\varphi \cos \vartheta)}{2 \theta_A \sin^2 \vartheta} \right] \]

Here, \( \vartheta = A/B \) is the dimensionless parameter of the problem, and quantities \( a_{11} \) and \( a_{13} \) are given by the formulas

\[ a_{11} = \cos \psi \cos \vartheta - \sin \psi \sin \varphi \cos \vartheta, \]
\[ a_{13} = \sin \psi \sin \vartheta. \]

The true anomaly \( \nu = \omega_0 t \) is taken as an independent variable, where \( \omega_0 \) is the angular velocity of the center of mass. The momenta \( p_\psi, p_\varphi, \) and \( p_\vartheta \) corresponding to coordinates \( \psi, \varphi, \) and \( \vartheta \) are reduced to the dimensionless form using the factor \( B \omega_0. \)

The equations of motion admit a particular solution on which

\[ \vartheta = \pi/2, \quad \varphi = 0, \quad p_\vartheta = p_\psi = 0, \]

while changes of variables \( \psi \) and \( p_\psi \) are described by the canonical equations with the Hamiltonian

\[ h = \frac{1}{2} (p_\psi - 1)^2 + \frac{3}{2} \sin^2 \psi. \]

This solution describes those planar motions of the satellite at which its middle or the largest axis of inertia is perpendicular to the plane of the orbit of the center of mass, i.e., the plane of a satellite-plate is perpendicular to the orbit plane.

It is worthy of note that the equations of motion admit also a particular solution describing the planar motions of the satellite at which its least axis of inertia is perpendicular to the orbit plane (the plate lies in the plane of orbit). The problem of orbital stability of planar periodic motions of this type was investigated in [6, 12].

If the value of energy constant \( h_0 < 3/2 \), the solution of the system of equations with Hamiltonian (2) has the following form

\[ \psi^*(\nu, k) = \arcsin[k \sin(\sqrt{3} \nu, k)], \]
\[ p_\psi^*(\nu, k) = 1 + \sqrt{3} k \cos(\sqrt{3} \nu, k), \quad k = \sin \psi_0 \]

and it describes planar pendulum oscillations with amplitude \( \psi_0 \) (\( \psi_0 < \pi/2 \)) and frequency \( \Omega = \pi \sqrt{3}/2 K(k) \).

Here and below, the complete elliptic integrals of the first and second kind are designated as \( K(k) \) and \( E(k) \), respectively.

In order to analyze the orbital stability of planar oscillations, let us apply the technique of papers [7, 8]. Following it, we make the canonical change of variables \( \psi, p_\psi \rightarrow w, l \) according to the formulas

\[ \psi = \psi^*(w/\Omega, k), \quad p_\psi = p_\psi^*(w/\Omega, k), \quad \Omega = \sqrt{3}/2 K(k) \]

in which \( k = k(l) \) is the function inverse to

\[ I = \frac{2 \sqrt{3}}{\pi} \left[ \frac{E(k)}{(1 - k^2)} K(k) \right]. \]

Let us note that the pair of canonical variables \( w, l \) in the unperturbed motion represents the variables action–angle. In these variables Hamilton function (2) has the form \( h = 3k^2(l)/2 \), and unperturbed motion is described by the relations \( I = I_0, w = \Omega v + w_0 \).

Let us introduce perturbations \( q_1, q_2, p_1, p_2, \) and \( r \)

\[ \varphi = q_1, \quad \vartheta = \pi/2 + q_2, \quad p_\psi = p_1, \]
\[ p_\vartheta = p_2, \quad l = I_0 + r \]

and go over to a new independent variable \( \tau = \Omega v \). The Hamiltonian of perturbed motion is written in the form

\[ H = H_2 + H_4 + O_6, \]

\[ H_2 = r + \tilde{h}_2(q_1, q_2, p_1, p_2, w), \]
\[ H_4 = \frac{1}{2 \Omega I_0} r^2 + r \psi(q_1, q_2, p_1, p_2, w) + \tilde{h}_4(q_1, q_2, p_1, p_2, w), \]
\[ \tilde{h}_2 = \frac{1}{\Omega} \left( \begin{array}{c} (1 - \theta_A)q_1^2 + 3 \frac{1}{2} \alpha_2 q_1 q_2 \\ \theta_A \end{array} \right) \]
\[ + \frac{1}{2 \Omega I_0} q_1^2 + \frac{1}{2 \Omega I_0} q_2^2 \sin^2 \psi \]
\[ \psi = \frac{1}{\Omega} \left( \begin{array}{c} (1 - \theta_A) \partial q_1^2 + \frac{3}{2} \alpha_2 q_1 q_2 \\ \theta_A \end{array} \right) \]
\[ + \frac{1}{2 \Omega I_0} q_1^2 + \frac{1}{2 \Omega I_0} q_2^2 \sin^2 \psi \]

\[ \Omega = \pi \sqrt{3}/2 K(k). \]

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