CONTROL THEORY

Inversion of Vector Delay Systems

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Abstract—We consider stationary linear vector systems with commensurable delays. We obtain sufficient conditions for the reducibility of such systems to canonical form with the extraction of null dynamics. A constructive algorithm for the reduction of a system to that form is presented. We suggest a method for estimating the unknown input for vector delay systems with given accuracy.

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1. INTRODUCTION. STATEMENT OF THE PROBLEM

The inversion problem that is the problem of online reconstruction of the unknown input of a system on the basis of measurements of its output is a classical inverse problem of control theory. Various aspects of this problem were studied in numerous papers [1–7], where various statements of the problem determined by the form of the system and the conditions imposed on the performance of the desired estimate were considered.

The inversion problem is of large practical value in control theory. Estimating unknown input signals (noises) permits one to solve both classical stabilization problems in the case of uncertainty and nonclassical stabilization, for example, the problem of simultaneous stabilization of families of dynamical systems [9–12] subjected to external disturbances.

In the present paper, we study the inversion problem for a system described by linear stationary differential equations with commensurable delays. Such a problem for a system with scalar input and output was considered in [7]. In the present paper, we generalize earlier-suggested approaches to systems with vector input and output.

Consider the formal statement of the problem. Consider the dynamical system

\[
\dot{x} = \sum_{i=0}^{k} A_i x(t - i\tau) + \sum_{i=0}^{k} B_i \xi(t - i\tau), \quad y = \sum_{i=0}^{k} C_i x(t - i\tau), \quad t \geq 0, \tag{1}
\]

where \(x(t) \in \mathbb{R}^n\) is the system state vector, \(y(t) \in \mathbb{R}^l\) is the measured output of the system, \(\xi(t) \in \mathbb{R}^m\) is the unknown input, \(A_i, B_i,\) and \(C_i\) are constant known matrices of the corresponding dimensions, \(\tau, 2\tau, \ldots, k\tau\) are constant (commensurable) delays, and \(k\tau\) is the maximum value of the delay in the state vector, input, and output. The initial functions \(x(\Theta)\) and \(\xi(\Theta)\) are defined for \(\Theta \in [-k\tau, 0]\) and are such that there exists a unique solution of system (1) for \(t \in (0, +\infty)\); however, these initial functions themselves are assumed to be unknown.

The problem is, given output measurements \(y(t)\) \((t \geq 0)\), to construct an estimate \(\hat{\xi}(t)\) such that \(|\hat{\xi}(t) - \xi(t)| < \varepsilon\) for a given \(\varepsilon > 0\) starting from some instant of time \(t^* > 0\). The estimate \(\hat{\xi}(t)\) should be formed online on the basis of current values \(y(t)\) and the values \(y(\Delta)\) for \(\Delta \in [0, t]\) (that is, “memory” of earlier output values).

For forthcoming considerations, it is convenient to use the delay operator \(d(f(t)) = f(t - \tau)\) (which commutes with the differentiation operator) when describing system (1). Then system (1) can be represented in the form

\[
\dot{x} = A(d)x + B(d)\xi, \quad y = C(d)x, \tag{2}
\]
occurring on the diagonal are invariant polynomials of the matrix $C$. A polynomial matrix $C$ be a unimodular matrix [i.e., $\text{det} C = 1$, where $C$ is a matrix of polynomials in $x$].

It was shown in [7] that a necessary condition for the invertibility of system (1) (i.e., for distinguishing inputs distinct modulo asymptotically decaying terms on the basis of output measurements to be possible in principle) is that the Rosenbrock matrix $R(s, \dot{x}) = \begin{bmatrix} sI - A(\dot{x}) & -B(\dot{x}) \\ C(\dot{x}) & 0 \end{bmatrix} \in \mathbb{C}^{(n+l) \times (n+m)}$

of the system, where $\dot{x} = e^{-s\tau}$, does not have unstable invariant zeros.\(^1\)

In the case of linear stationary delay-free systems, the invariant zeros of the Rosenbrock matrix define the null dynamics of the system, that is, the character of its motion along the manifold $y(t) \equiv 0$. To solve the inversion problem, it is convenient to pass to a representation in which $y(t)$ is part of the state vector.

2. REDUCTION OF THE SYSTEM TO A SPECIAL FORM

We pass from the state vector $x(t)$ to a vector $\begin{bmatrix} x'(t) \\ y(t) \end{bmatrix}$, where $y(t) \in \mathbb{R}^l$ is the measured output and $x'(t) \in \mathbb{R}^{n-l}$ is the remaining part of the state vector. Since $y(t) = C(d)x(t)$, it follows that this coordinate transformation is given by a matrix $T(d) = \begin{bmatrix} T'(d) \\ C(d) \end{bmatrix}$, where $T'(d) \in \mathbb{R}^{(n-l) \times n}[d]$ is a matrix of polynomials in $d$. For this change of variables to be invertible, it is necessary that $T'(d)$ be a unimodular matrix [i.e., $\text{det} T'(d) = \text{const} \neq 0$]. Thus, we arrive at the following problem: given a polynomial matrix $C(d) \in \mathbb{R}^{l \times n}[d]$, find a polynomial matrix $T'(d)$ such that the matrix $T'(d)$ is unimodular.

Every polynomial matrix $C(d)$ can be reduced to Smith form [8, pp. 143–144]; i.e., for $C(d)$ there exist unimodular matrices $L(d) \in \mathbb{R}^{l \times l}[d]$ and $R(d) \in \mathbb{R}^{n \times n}[d]$ such that

$$L(d)C(d)R(d) = \tilde{C}(d), \quad \tilde{C}(d) = (0, \hat{C}(d)),$$

where $\hat{C}(d) = \text{diag}([\hat{C}_1(d), \ldots, \hat{C}_l(d)]) \in \mathbb{R}^{l \times l}$ is a diagonal $l \times l$ matrix. The polynomials $\hat{C}_i(d)$ occurring on the diagonal are invariant polynomials of the matrix $C(d)$, and $\hat{C}_i(d)$ is a divisor of $C_{i+1}(d)$ ($i = 1, \ldots, l-1$).

Let $D_i(d)$ be the greatest common divisor of all $i$th-order minors of $C(d)$. Then we have the representation

$$\hat{C}_i(d) = \frac{D_i(d)}{D_{i-1}(d)}, \quad D_0(d) = 1.$$

Obviously, a necessary condition for the existence of a matrix $T'(d)$ is that the diagonal matrix $\hat{C}(d)$ in the Smith form of the matrix $C(d)$ is the identity matrix, i.e., that $\hat{C}_i(d) = 1$ for all $i = 1, \ldots, l$. Since $\hat{C}_i(d)$ is divisible by all entries of $\hat{C}_i(d)$, it follows that $\hat{C}(d)$ is the identity matrix if and only if $\hat{C}_i(d) = 1$. Since $\hat{C}_i(d) = D_i(d)/D_{i-1}(d)$, for this it suffices that $D_i = 1$, i.e., that the greatest common divisor of all minors of maximum order $l$ of the matrix $C(d)$ is the polynomial equal to unity.

This condition is also necessary. Indeed, if $\hat{C}_i(d) = 1$, then $\hat{C}_i(d) = 1$, $i = 1, \ldots, l$; therefore, $D_i(d) = D_{i-1}(d)$, $D_{i-1}(d) = D_{i-2}(d)$, $\ldots$, $D_1(d) = D_0(d) = 1$; i.e., $D_i(d) = 1$. We have thereby proved the following assertion.

\(^1\) An invariant zero is a number $s^* \in \mathbb{C}$ such that $\text{rank} R(s^*, e^{-s^*\tau}) < m + n$.  

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