Observe the stationarity hybrid discrete-continuous dynamical systems. Necessary and sufficient observability conditions expressed directly via the system parameters are derived. We consider linear observability problems and the dual controllability and reachability problems. The problem of computing the minimum number of inputs for which the system has a given observability is discussed. An example illustrating the results is presented.

**Statement of the Problem**

Consider a linear hybrid discrete-continuous system of the form

\[ \dot{x}_1(t) = A_{11}x_1(t) + A_{12}x_2(\mathcal{K}h), \quad t \in [\mathcal{K}h, \mathcal{K}h + h), \]

\[ x_2(\mathcal{K}h + h) = A_{21}x_1(\mathcal{K}h) + A_{22}x_2(\mathcal{K}h), \quad k = 0, 1, 2, \ldots, \]

with the initial conditions of the form

\[ x_1(0) = x_1(+0) = x_{10}, \quad x_2(0) = x_{20}, \]

The rapid development of computing facilities and the robotization of production processes pose new challenges for the design of modern automated control systems. In this connection, we note the increasing role of mathematical modeling for a more adequate description of production systems with regard of their specific features, for example, the presence of continuous and discrete variables, deterministic and random inputs, logical (Boolean) variables, etc., which often leads to mathematical models in the form of hybrid dynamical systems. However, note that there are various interpretations of the notion “hybrid systems” (see the papers [1–11] and the bibliography therein). From our viewpoint, hybridity implies inhomogeneity in the nature of the considered process or in methods of its analysis.

Robotization and, as a consequence, computerization of production processes stimulate studies of discrete systems [12]; in this connection, hybrid discrete-continuous systems [10, 11, 13], in which continuous and discrete inputs exist simultaneously (for example, one main dynamics of the system is described by continuous processes, and control and/or observing devices are described by discrete processes), become increasingly topical.

In the present paper, we consider hybrid discrete-continuous systems, which can be treated in the general case as continuous systems acted upon by discrete controllers. Such systems can be used in the investigation of various quantized [3] dynamical systems as well as systems with impulse actions [14, 15].

On the whole, the present paper continues the studies of main problems of qualitative theory of control and observation in various classes of hybrid dynamical systems, in particular, hybrid differential-difference systems (e.g., see [7–9]) and is directly close to the papers [10, 11] in which stability, stabilizability, controllability, and reachability problems were considered for discrete-continuous systems. In what follows, we study the observability problem for such systems.
and with the output
\[ y(kh) = B_1 x_1(kh) + B_2 x_2(kh), \quad k = 0, 1, 2, \ldots, \]
where 0 < h is the quantization step, \( x_1(t) \in \mathbb{R}^{n_1}, x_2(t) \in \mathbb{R}^{n_2}, y(kh) \in \mathbb{R}^m; \) and \( A_{ij} \) and \( B_i, \)
\( i = 1, 2, \) are constant matrices of the corresponding sizes.

A solution \( x_1(t) = x_1(t, x_{10}, x_{20}), t \geq 0, x_2(kh) = x_2(kh, x_{10}, x_{20}), k = 0, 1, 2, \ldots, \) of system (1), (2) with the initial conditions (3) is defined as an \( n_1 \)-vector function \( x_1(t) \) that is differentiable for \( t \neq kh \) and continuous for \( t \geq 0 \) and an \( n_2 \)-vector function \( x_2(kh), k = 0, 1, 2, \ldots, \) both of which satisfy Eqs. (1) and (2) and the initial conditions (3). The right derivatives are used in Eq. (1) at the points \( t = kh, k = 0, 1, \ldots \)

By integrating system (1), (2) step by step, we find that there exists a unique solution \( x_1(t, x_{10}, x_{20}), t \in [kh, kh + h], x_2(kh, x_{10}, x_{20}), k = 0, 1, 2, \ldots, \) of the initial value problem (3); moreover, unlike differential-difference systems in symmetric form [9], whose state space is infinite-dimensional, the state space of system (1), (2) coincides with \( \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \) and is finite-dimensional. Therefore, the pair \( (x_1(t, x_{10}, x_{20}), x_2(kh, x_{10}, x_{20})) \) is treated as the current state of system (1), (2) at time \( t \in [kh, kh + h] \).

**Definition 1.** Let \( y(kh) = y(kh, x_{10}, x_{20}), k = 0, 1, 2, \ldots, \) be the output (4) generated by the initial state \( x_{10}, x_{20} \). Then system (1), (2) with the output (4) is said to be:

1. **Strongly completely observable (SCO)** if, on the basis of measurements of output functions \( y(kh) \) for \( k = k_1, k_1 + 1, \ldots \) with some positive integer \( k = k_1 \) (which is not given in advance), one can distinguish the initial states of the system generating those functions; i.e.,

\[
(x_{10}, x_{20}) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}, \quad (\tilde{x}_{10}, \tilde{x}_{20}) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}, \quad y(kh, x_{10}, x_{20}) \equiv y(kh, \tilde{x}_{10}, \tilde{x}_{20}),
\]

\[
k = k_1, k_1 + 1, \ldots, \quad k_1 \in \mathbb{N} \implies x_{10} = \tilde{x}_{10}, \quad x_{20} = \tilde{x}_{20}.
\]

2. **Completely observable (CO)** if the conditions \( x_{10} = \tilde{x}_{10} \) and \( x_{20} = \tilde{x}_{20} \) are satisfied for arbitrary \( (x_{10}, x_{20}) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \) and \( (\tilde{x}_{10}, \tilde{x}_{20}) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \) such that \( y(kh, x_{10}, x_{20}) \equiv y(kh, \tilde{x}_{10}, \tilde{x}_{20}), k = 0, 1, 2, \ldots, \)

3. **Weakly completely observable (WCO)** if the relations

\[
(x_{10}, x_{20}) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}, \quad (\tilde{x}_{10}, \tilde{x}_{20}) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2},
\]

\[
y(kh, x_{10}, x_{20}) \equiv y(kh, \tilde{x}_{10}, \tilde{x}_{20}), \quad k = 0, 1, 2, \ldots,
\]

imply that

\[
x_1(t, x_{10}, x_{20}) = x_1(t, \tilde{x}_{10}, \tilde{x}_{20}), \quad t > 0,
\]

\[
x_2(kh, x_{10}, x_{20}) = x_2(kh, \tilde{x}_{10}, \tilde{x}_{20}), \quad k > 0, \quad k \in \mathbb{N}.
\]

4. **Strongly terminally observable (STO)** if, on the basis of measurements of the output functions \( y(kh) \) for \( k = k_1, k_1 + 1, k_1 + 2, \ldots \) with some positive integer \( k = k_1 \) (which is not given in advance), one can distinguish the corresponding current states of the system at times \( t = kh, k = k_2, k_2 + 1, k_2 + 2, \ldots, \) where \( k_2 \) is some sufficiently large positive integer; i.e., the relations

\[
(x_{10}, x_{20}) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}, \quad (\tilde{x}_{10}, \tilde{x}_{20}) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2},
\]

\[
y(kh, x_{10}, x_{20}) \equiv y(kh, \tilde{x}_{10}, \tilde{x}_{20}), \quad k = k_1, k_1 + 1, k_1 + 2, \ldots, \quad k_1 \in \mathbb{N},
\]

imply the existence of \( k_2 \in \mathbb{N} \) such that

\[
x_i(kh, x_{10}, x_{20}) \equiv x_i(kh, \tilde{x}_{10}, \tilde{x}_{20}), \quad k \geq k_2, \quad k \in \mathbb{N}, \quad i = 1, 2.
\]

5. **Weakly terminally observable (WTO)** if the relations

\[
(x_{10}, x_{20}) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}, \quad (\tilde{x}_{10}, \tilde{x}_{20}) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2},
\]

\[
y(kh, x_{10}, x_{20}) \equiv y(kh, \tilde{x}_{10}, \tilde{x}_{20}), \quad k = 0, 1, 2, \ldots,
\]