Laws of Flow with a Limiting Gradient in Anisotropic Porous Media

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Received June 22, 2009

Abstract—The equations of viscoplastic fluid flow through a porous medium are written for all types of anisotropy. It is shown that in anisotropic media the flows with a limiting gradient are characterized by two material tensors: the tensor of permeability (flow resistance) coefficients and the tensor of limiting gradients. A complex of laboratory measurements for determining the tensors of permeability coefficients and limiting gradients is considered for all types of anisotropic media. It is shown that the tensors of permeability coefficients and limiting gradients are coaxial. Conditions of flow onset and fluid flow laws are formulated for media with monoclinic and triclinic symmetries of flow characteristics.

DOI: 10.1134/S0015462810020079

Keywords: viscoplastic fluid flow through a porous medium, anisotropy, tensor of limiting (initial) gradients, flow onset condition, surface of limiting (initial) gradients.

Drilling muds and anomalous oils often manifest non-Newtonian properties describable by the Bingham-Schweyoff rheological equations and can be represented as viscoplastic fluids with a limiting gradient [1, 2]. In [3, 4], we considered the equations of fluid flow in media with transversely isotropic and orthotropic flow characteristics. Practically, however, natural hydrocarbon reservoirs are encountered with a more complicated, monoclinic and triclinic, symmetry of flow characteristics [5, 6]. Therefore, the formulation and analysis of flow laws for the media with monoclinic and triclinic symmetries of flow characteristics is of practical, not only theoretical, interest.

1. EQUATIONS OF FLOWS WITH A LIMITING GRADIENT IN ANISOTROPIC POROUS MEDIA

Equations of flows through a porous medium with a limiting gradient are commonly written in the form solved for the fluid velocity vector and have the form [2]:

\[ w_i = -\frac{k_{ij}}{\mu} \left( \delta_{ji} - \frac{\gamma_{ji}}{|\nabla p|} \right) \nabla_i p, \]  

(1.1)

where \( w_i \) are the fluid velocity components, \( k_{ij} \) and \( \gamma_{ji} \) are the components of the tensors of permeability coefficients and limiting (initial) gradients, respectively, \( \delta_{ji} \) is the Kronecker delta, \( \nabla_i p \) are the pressure gradient vector components, \( |\nabla p| \) is the magnitude of the pressure gradient vector, and \( \mu \) is the viscosity. In Eq. (1.1) and in what follows, unless otherwise specified, summation over recurring Latin indices is assumed.

We will rewrite the equation of fluid flow with a limiting gradient in the form solved for the pressure gradient vector components. For this purpose, we will multiply equality (1.1) by the symmetric second-rank tensor \( r_{ij} \) inverse to the tensor of permeability coefficients, which is called the tensor of flow resistance coefficients. Finally, we obtain
\[ \mu r_{ij} w_j = -\nabla_i p + \gamma_j n_j, \quad \nabla_j p = |\nabla p| n_j, \tag{1.2} \]

where \( n_j \) are the components of the unit vector determining the direction of the pressure gradient vector.

In Eq. (1.2) the representation of the tensors of flow resistance coefficients and limiting gradients depends on the type of anisotropy and for all symmetry groups is written in the maximally general form in [7]. In the general case, the symmetries of the tensors \( k_{ij} \) and \( \gamma_{ij} \) may be different. For example, the tensor of permeability coefficients may be anisotropic and the tensor of limiting gradients isotropic, and vice versa, or both tensor may be anisotropic with different symmetry (anisotropy) types. In further considerations we will assume that both tensors are of the same symmetry (anisotropy) type.

For second-rank tensors there are only four different types of anisotropic characteristics. All the four possible types of symmetric second-rank tensors determining the material characteristics can be represented in the form:

\[ r_{ij} = r_1 \delta_{ij} + r_2 B_{ij}, \]
\[ r_{ij} = r_1 \delta_{ij} + r_2 B_{ij} + r_3 D_{ij}, \]
\[ r_{ij} = r_{11} a_i a_j + r_{22} c_i c_j + r_{33} b_i b_j + r_{12} (a_i c_j + a_j c_i), \]
\[ r_{ij} = r_{11} a_i a_j + r_{22} c_i c_j + r_{33} b_i b_j + r_{12} (a_i c_j + a_j c_i) + r_{23} (c_i b_j + c_j b_i) + r_{13} (a_i b_j + a_j b_i), \]

where \( r_\alpha \) and \( r_{\alpha\beta} \) are invariant components of the flow resistance tensors and \( B_{ij}, D_{ij}, a_i, b_i, \) and \( c_i \) are basis tensors defining and determining the symmetry of the physical characteristics of anisotropic media, the last three tensors being representable as dyads formed by the basis vectors of the crystal coordinate system [8].

The first equality in relations (1.3) determines transversally isotropic and the second orthotropic material characteristics. The last two equalities determine monoclinic and triclinic characteristics, respectively. Similarly, correct to the replacement of \( r_\alpha \) and \( r_{\alpha\beta} \) by \( \gamma_\alpha \) and \( \gamma_{\alpha\beta} \), the tensors of limiting gradients can be determined.

Since the Neumann principle determines the relationship between the symmetry of physical characteristics and the geometric symmetry, each anisotropy type can be associated with a geometric figure. Transversally isotropic properties correspond to a rectangular parallelepiped with a square base, orthotropic properties to a rectangular parallelepiped with a rectangular base, and monoclinic and triclinic properties to skew-angular parallelepipeds. For monoclinic properties only one of the angles is acute, whereas for triclinic properties all the three are.

Substituting tensors (1.3) in equality (1.2), the equations of fluid flows with a limiting gradient can be obtained for all types of anisotropic characteristics. However, equality (1.2) holds only if the flow onset conditions are satisfied. For isotropic flow characteristics the flow onset condition takes the simplest form and can be reduced to the inequality \( |\nabla p| > \gamma \) [1]. For anisotropic flow characteristics, as we will show below, the flow onset condition leads to a direction-dependent inequality. Analysis of the inequalities that determine flow onset conditions shows that one-, two- and three-dimensional fluid flows with a limiting gradient are possible [6] and each case corresponds to its own specific flow onset condition and fluid flow equation.

Before analyzing the flow onset conditions for media with monoclinic and triclinic properties, we will consider model problems of determining the fluid flow characteristics in anisotropic media.

2. MODEL PROBLEMS OF FINDING MATERIAL CHARACTERISTICS DETERMINED BY SYMMETRIC SECOND-RANK TENSORS

In modern crystallography, for the laboratory determination of material characteristics determined by symmetric second-rank tensors two model problems are usually considered. In terms of the theory of flows through porous media these problems can be formulated as the problem of flow across a thin plate and the