Rapid Evaporation and Condensation of Gas between Two Surfaces in the Limit of Low Values of Knudsen Number

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Abstract—One-dimensional plane, cylindrical, and spherical flows are considered, which arise in the case of rapid evaporation and condensation of gas. It is demonstrated that the main similarity parameter is $\bar{p}_{ev}$, i.e., the ratio of saturated vapor pressures at temperatures of “hot” and “cold” surfaces. The boundaries of the regions of limiting steady-state flows are investigated. The existence is demonstrated of the optimal value of the ratio of surface radii, at which the flow assumes the limiting mode at minimal value of $\bar{p}_{ev}$. The effect of polyatomicity of gas is considered.

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INTRODUCTION

Evaporation and condensation flows often arise in present-day technological processes, when intensive evaporation from heated region occurs accompanied by intensive condensation on cooled regions. In so doing, a layer with a thickness of several free paths arises in the gas flow in the vicinity of interface; no translational equilibrium is present in this layer. The relations on the boundary of this layer (referred to as Knudsen layer) are the boundary conditions for hydrodynamic equations. The Mach number $M$ appears in these relations as a parameter for whose determination the flow as a whole must be analyzed. This study is devoted to consideration, in a one-dimensional formulation, of flows arising in the case of evaporation and condensation of matter.

FORMULATION OF THE PROBLEM

Plane Geometry

We will consider the problem on steady-state evaporation and condensation of matter from a plane surface at temperature $T_1$ to a parallel surface at temperature $T_2 < T_1$ [1–3].

The evaporation and condensation are assumed to be rapid, the Knudsen number is assumed to be much less than unity, and the condensation coefficient is taken to be unity. The relations on the boundaries of arising Knudsen layers have the form [4]

1) in the case of evaporation,

$$\frac{P}{P_{ev}(T_1)} = p_1(M, k) ; \quad \frac{T}{T_1} = \tau_1(M, k) ; \quad (1)$$

and

2) in the case of condensation,

$$\frac{P}{P_{ev}(T_2)} = p_2(M, k, \tau_2) . \quad (2)$$

Here, $P$ is the gas pressure in the flow, $P_{ev}$ is the saturated vapor pressure at preassigned temperature, $T$ is the flow temperature, $k$ is the adiabatic exponent, and $\tau_2 = T/T_2$.

Both the moments methods [5, 6] and the method of direct statistical simulation [7, 8] were used to investigate relations (1) and (2) with the condensation and evaporation coefficients equal to unity. Good agreement of the obtained results was observed in a wide range of parameters, which enables one to use moments solutions as approximations.

Therefore, the Mach number of flow is determined from the equations

$$\frac{p_2(M, k, \tau_2)}{p_1(M, k)} = \frac{P_{ev}(T_1)}{P_{ev}(T_2)} ,$$

$$\tau_2 = \tau_1(M, k) \frac{T_1}{T_2} \quad (3)$$

The dependence of $M$ on $\bar{p}_{ev} = P_{ev}(T_1)/P_{ev}(T_2)$, obtained by solving Eq. (3) at $k = 5/3$, is given in Fig. 1.
for different values of the ratio $\tau = T_1/T_2 > 1$. The analytical dependences $p_1(M, k)$, $p_2(M, k, \tau_2)$, and $\tau_1(M, k)$ were used, which were obtained in [5, 9]. At $T_1/T_2 > 1$, they agree well with the results of numerical simulation of evaporation and condensation of monatomic gas,

$$p_1(M, k) = \sqrt{\tau_1} \left[ \exp(m^2) \text{erfc}(m) \left( \frac{m^2 + \frac{1}{2}}{\sqrt{\pi}} \right) + \frac{1}{2}g(|m|) \right],$$

$$\tau_1(M, k) = \left[ \frac{-(k-1)\sqrt{\pi}|m|}{2(k+1)} + \sqrt{\left[ \frac{(k-1)\sqrt{\pi}m^2}{2(k+1)} \right]^2 + 1} \right],$$

$$p_2(M, k, \tau_2) = \frac{(b_1a_{22}a_{33} - b_1a_{23}a_{32} - b_2a_{12}a_{33} + b_2a_{13}a_{32} + b_3a_{12}a_{32} - b_3a_{13}a_{22})\tau_2}{(a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{13}a_{21}a_{32} + a_{12}a_{31}a_{23} - a_{13}a_{31}a_{22})},$$

where

$$a_{11} = m\sqrt{2}\sqrt{\tau_2}; \quad a_{12} = \frac{\sqrt{T_b}g(m_2)\exp[-(m_2)^2]}{\sqrt{2\pi}};$$

$$a_{13} = \frac{g(m_1)\exp[-(m_1)^2]}{2\sqrt{2m_1\pi}};$$

$$a_{21} = \tau_2(2m^2 + 1);$$

$$a_{22} = T_b\left[ \frac{m_2^2}{\sqrt{\pi}} \left( g(m_2)\exp[-(m_2)^2] \right) - \frac{1}{2}\text{erfc}(m_2) \right];$$

$$a_{23} = \frac{1}{2}\left[ \frac{m_1^2}{\sqrt{\pi}} \left( g(m_1)\exp[-(m_1)^2] \right) - \frac{1}{2}\text{erfc}(m_1) \right];$$

$$a_{31} = \frac{1}{2}m_1^2(\tau_2)^3\sqrt{2}(2m^2 + 5),$$

$$a_{32} = -T_b\left[ \frac{\sqrt{T_b}}{2\pi} \left( (m_2)^2 + \frac{5}{2} + \frac{3k}{2k-2} \left( 1 - \frac{\tau_2}{T_b} \right) \right) \right]$$

$$\times \left( 1 - g(m_2) \right) - (m_2)^2 - 2 \exp[-(m_2)^2],$$

$$a_{33} = \frac{1}{2\sqrt{2}} \left( m_1^3 \right)^2 \left[ g(m_1) \left[ \left( 1 - \frac{(5-3k)}{2k-1} \right) \frac{\tau_2\pi}{\left( m_1 \right)^2} + \frac{k}{(k-1)} \right] - \frac{1}{2} \right],$$

$$b_1 = \frac{1}{\sqrt{2\pi}}; \quad b_2 = \frac{1}{2};$$

$$b_3 = \left( \frac{1}{\sqrt{2\pi}} \right) \left[ 2 + \frac{5-3k}{2(k-1)}(1 - \tau_2) \right].$$

$$g(m) = 1 - \sqrt{\pi m}\text{erfc}(m)e^{-m^2};$$

$$\text{erfc}(m) = \frac{2}{\sqrt{\pi}} \int_m^\infty e^{-t^2} dt,$$

$$T_b = 0.012 + \tau_2 \times 0.988$$

$$\left[ \frac{1 + \tau_2}{2} - (0.012 + \tau_2 \times 0.988) \right](1 - M)^2,$$

$$m = -\frac{\sqrt{3}}{\sqrt{2}M}; \quad m_1 = \frac{3}{3\pi - 2}; \quad m_2 = m \sqrt{\frac{\tau_2}{T_b}}.$$

It follows from Eq. (3) that the Mach number is a function of two parameters, namely, $\bar{p}_e$ and $\tau$ depen-

![Fig. 1. The Mach number of flow for one-dimensional plane flow as a function of $\bar{p}_e$ at different wall temperatures: (1–3) $k = 4/3$; (4–6) $k = 7/5$; (7–9) $k = 5/3$; for (1, 4, 7) $\tau = 4$; for (2, 5, 8) $\tau = 3$; for (3, 6, 9) $\tau = 2$.](image-url)