Efficiency of Heat Transfer in Recuperative Heat Exchangers with High-Speed Gas Flows at Low Prandtl Numbers

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Abstract—The possibility to estimate the efficiency of recuperative heat exchangers is shown. The estimation takes into account the effect of gas dynamic energy separation using the known parameters: the efficiency of the recuperator, the number of transfer units, and the specific surface area expressed through recovery temperatures. The dependences of the recuperator parameters on the reduced rates of heat-transfer agents are calculated at Prandtl numbers considerably different from unity.

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INTRODUCTION

The development and production of devices for highly efficient energy transfer between gaseous heat-transfer agents having low Prandtl numbers and moving with significant rates evoke considerable interest in recent years. The operation of these devices is based on violation of the Reynolds analogy caused by anomalously low Prandtl number \( \text{Pr} \), which characterizes the physical properties of a heat-transfer agent, being the product of dynamic viscosity \( \mu \) by isobaric heat capacity \( c_p \) referred to heat conductance, and a significant difference in the rates of cold and hot heat-transfer agents [1–4]. An additional effect can be obtained, most likely, by using high-speed two-phase heat-transfer agents [5, 6].

This effect is especially pronounced in heat-transfer agents representing mixtures of heavy and light gases, for example, helium with xenon or hydrogen with xenon. A series of recent works was devoted to the energy separation of gases by the method of gas dynamic temperature stratification proposed by Academician A.I. Leont’ev [7–11]. The schemes of moving heat-transfer agents in recuperative heat exchangers and the retardation temperatures \( \lambda_{1,2} \) and reduced rates \( \lambda_{1,2} \) of the cold and hot heat-transfer agents, respectively, are shown in Fig. 1.

To estimate the efficiency of heat-transfer processes, let us use the parameters applied in the theory of heat-exchange devices, among which are the efficiency of a heat-transfer agent \( \varepsilon \), the average logarithmic temperature head \( \Delta T \), the specific surface area \( f \), and the number of transfer units \( NTU \).

The purpose of this work is to evaluate the possibility of using these parameters for determination of the efficiency of recuperative heat exchangers with high-speed heat-transfer agents that have low Prandtl numbers and to reveal specific regularities of their application.

RESTATEMENT BETWEEN THE HEAT-TRANSFER AGENT VELOCITIES AND THE QUANTITY OF HEAT TRANSFERRED IN A RECUPERATIVE HEAT EXCHANGER

The efficiency of a heat exchanger \( \varepsilon \) plays an important role among the parameters that characterize the perfection of heat exchangers and their mass and size characteristics. Efficiency \( \varepsilon \) is defined as the ratio of the really transferred heat to the maximum possible heat in the countercurrent scheme of motion of heat-transfer agents and an infinitely high surface of heat exchange [1].

\[
\varepsilon = \frac{Q}{Q_{\text{max}}} = \frac{W_i \left( T_i'' - T_i' \right)}{W_{\text{min}} \left( T_i' - T_i'' \right)} = \frac{W_i \left( T_i' - T_i'' \right)}{W_{\text{min}} \left( T_i' - T_i'' \right)},
\]

where \( W_i = c_p \dot{m}_i \) is the total heat capacity of the flow of the \( i \)th heat-transfer agent (1 denotes the hot heat-transfer agent, and 2 is the cold one), being the product of the heat capacity at a constant pressure \( c_p \) by the consumption of the \( i \)th heat-transfer agent; \( W_{\text{min}} = \min(W_1; W_2) \) is the minimum heat capacity of the flow; \( T_i' \) and \( T_i'' \) are the temperatures of the hot heat-transfer agent at the inlet and outlet, respectively; \( T_i' \) and \( T_i'' \) are the temperatures of the cold heat-transfer agent at the inlet and outlet, respectively.

The temperature head in an arbitrary cross section of the recuperator is determined by the difference between the recovery temperatures \( T_{\text{WT}} \) on both sides of the heat-exchange surface.
They are nearly equal to the retardation temperatures at low flow rates and (or) the Prandtl number close to unity

In the case of using gaseous heat-transfer agents with the Prandtl numbers substantially lower than unity (0.15–0.25), at elevated rates of heat-transfer agent motion, the recovery temperature of the heat-transfer agent can be determined from the dependence [12]

\[ \Delta T = T_{w1} - T_{w2} \]

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\[ T_{w1,2} = T_1^* \left[ 1 + (n_{1,2} - 1) \frac{\gamma - 1}{\gamma + 1} \lambda_{i,2} \right], \]

where \( \lambda_i = c_i/a_i \) is the reduced rate equal to the ratio of flow rate \( c_i \) to critical rate \( a_c \); \( \gamma \) is the adiabatic index, and \( n_{1,2} \) is the coefficient of temperature recovery.

Then, the maximum quantity of heat that can be transferred in a countercurrent recuperator will be written in the form

\[ Q_{\text{max}}^c = W_{\min} \left[ T_1^* \left[ 1 - (1 - n_1) \frac{\gamma - 1}{\gamma + 1} \lambda_1 \right] \right. \]

\[ \left. - T_2^* \left[ 1 - (1 - n_2) \frac{\gamma - 1}{\gamma + 1} \lambda_2 \right] \right], \]

Referring this value to the maximum heat quantity transferred in a classical (low-speed, or at the Prandtl number close to unity) countercurrent recuperator

\[ Q_{\text{max}}^0 = W_{\min} \left( T_1^* - T_2^* \right), \]

we obtain

\[ \frac{Q_{\text{max}}^c}{Q_{\text{max}}^0} = 1 - \frac{(1 - r) \gamma - 1}{\gamma + 1} \frac{\gamma - 1}{\gamma + 1} \frac{\lambda_1^2 - \lambda_2^2}{\lambda_1^2}, \]

where \( \gamma = T_1^*/T_2^* \) is the ratio of temperatures of the heat-transfer agents at the inlet of the device introduced for simplification.

Similar calculations for the concurrent flow give the dependence

\[ \frac{Q_{\text{max}}^c}{Q_{\text{max}}^0} = \frac{1}{1 + W_{\text{f}}} \left[ 1 - \frac{(1 - r) \gamma - 1}{\gamma + 1} \frac{\gamma - 1}{\gamma + 1} \frac{\lambda_1^2 - \lambda_2^2}{\lambda_1^2} \right]. \]

where \( W_{\text{f}} \) is the ratio of total heat capacities of the heat-transfer agents.

The influence of the reduced rate on the relative maximum transferred quantity of heat at equal total heat capacities of the heat-transfer agent flows \( W_{\text{f}} = 1 \), the ratio of retardation temperatures at the inlet of the heat exchanger \( \gamma = 2 \), the adiabatic index \( \gamma = 5/3 \), and the Prandtl number \( \text{Pr} = 0.25 \) for the laminar \( (r = \sqrt{\text{Pr}} = 0.5) \) and turbulent \( (r = \sqrt[3]{\text{Pr}} = 0.63) \) flows is shown in Fig. 2.

It follows from the plots in Fig. 2 that, when using heat-transfer agents with low Prandtl numbers in the case of high rates of the cold heat-transfer agent and low rates of the hot one, the maximum quantity of transferred heat is higher than that in the classical case, and this influence for the turbulent flow is weaker than for the laminar flow. For the maximum value of the reduced rate of the cold heat-transfer agent \( \lambda_2 = \lambda_{\text{max}} = 1/\sqrt{(\gamma - 1)} \) and negligibly low value of

**Fig. 2. Influence of the reduced rate of the cold heat-transfer agent on the relative maximum quantity of heat transferred in the recuperator: (1) countercurrent (laminar flow), (2) countercurrent (turbulent flow), (3) concurrent (laminar flow), and (4) concurrent (turbulent flow).**

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**Fig. 1. Schemes of the heat-exchange agent motion in recuperative heat-exchange devices: (a) concurrent, (b) countercurrent, and (c) cross current.**