NUCLEATION AND GROWTH OF A GAS BUBBLE IN MAGMA

M. N. Davydov

Abstract: The dynamics of a “collective” gas bubble in the magma melt during its decompression was numerically studied on the basis of a complete mathematical models of an explosive volcanic eruption. It is shown that the bubble size distribution obtained for the nucleation process has one peak, which allows considering a “collective” bubble. The main stages of bubble growth due to gas diffusion and changes in the viscosity of the medium are determined. It is shown that the high viscosity of the melt makes possible the transition from the Rayleigh equation to a simpler relation for the radial velocity of the bubble.

Keywords: explosive volcanic eruption, decompression, bubble dynamics, viscosity.

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INTRODUCTION

The study of volcanic eruptions and various processes occurring within the volcano involves great difficulties. However, field observations and the known results of modeling allow assuming that the process of volcanic eruption is largely determined by dynamic changes in the magma melt state [1].

One of the main factors determining volcanic eruptions [2] is changes in the viscosity of the magma melt which substantially depend on the concentration of water dissolved in it. It is known that in the initial state, magma contains melt components such as carbon dioxide, sulfur, and water. The mass concentration of water may be 5–7% of the mass of magma. Decompression during eruptions causes magma to degas substantially and gas bubbles to form in the magma, while its viscosity can increase by a few orders of magnitude (in the range of $10^2$ to $10^{12}$ Pa·s).

The kinetics of nucleation and growth of gas bubbles in gas-saturated magma melts during decompression were experimentally studied in [3–6]. In particular, it has been shown that the rate of nucleation and the finite number of bubbles formed strongly depend on the rate of decompression and magma composition. It has been established that two stages can be distinguished in the process of bubble growth: exponential growth due to magma decompression and further slow diffusional growth. It should be noted that high viscosity actually leads to the cessation of the processes occurring in the magma. Thus, at a viscosity of more than $10^8$ Pa·s, the magma is not completely degassed, in spite of decompression up to a pressure of 50 MPa [6].

A large number of papers is devoted to the creation of various models of bubble growth based on different approaches. It has been assumed [3, 7], in particular, that each bubble is surrounded by a spherical cell, whose boundaries are subject to decompression conditions. This approach to accounting for viscous effects on the bubble boundary was used in [8], where a bubble surrounded by a thin layer of a viscous liquid was considered. A model taking into account thermal effects was proposed in [9]. It was shown in [10] that the results of calculations for this model are in better agreement with the experimental data obtained in the wide range of initial mass of gas concentration than those for other models. The effect of diffusion on the dynamics of the bubble was studied in [11]. In accordance with the above approaches (despite their differences), among the magma parameters the viscosity of the medium plays a decisive role in the eruption process [12].
It should be noted that in many studies (see [7, 9, 10]), magma decompression was assumed to be instantaneous or following some simple law. Of course, under real conditions where a decompression wave propagates along the volcanic vent, this assumption is not satisfied.

A number of unresolved questions remains, however, despite extensive research of the processes occurring during the eruption of volcanoes. In particular, estimates show that with the total degassing of a magma in which the initial mass fraction of water was 5–7%, the medium should almost entirely consist of bubbles; yet the porosity of the lava rock is 10–30%. There are hypotheses linking various volcanic phenomena to the dynamics of bubbles in the magma melt. For example, L’Heureux [13] studied the effect of the rate of bubble growth on the frequency of volcanic activity. The mechanism of formation of volcanic ash may be based on the coalescence of gas bubbles, which leads to the need to determine their number and characteristic sizes.

The purpose of this paper is a detailed numerical study of bubble nucleation and growth during passage of a decompression wave along a magmatic column.

FORMULATION OF THE PROBLEM

A vertical column of a gas-saturated magma melt of height $H$ under gravity borders a magma chamber at the bottom, in which the pressure is equal to $p_{ch}$. Above the column is an external medium at pressure $p_0$. The $z$ axis is directed vertically upward, and the origin is located at the boundary between the chamber and the column [14]. The initial pressure in the chamber–magma column system corresponds to the pressure in the magma chamber taking hydrostatics into account: $p_i(z) = p_{ch} - \rho_0 g z$ ($\rho_0$ is the magma density and $p_{ch}$ is the pressure at $z = 0$).

It is assumed that gas dissolved in the magma initially has equilibrium concentration $C_{eq}$, whose dependence on the pressure $p$ is defined by the Henry’s law. For water dissolved in the magma melt, this dependence has the form [15]

$$C_{eq}(p) = K_H \sqrt{p},$$

where $K_H$ is the Henry’s constant. Accordingly, the dependence of the initial concentration $C_i$ of the gas dissolved in the magma on $z$ is defined by $C_i(z) = C_{eq}(p_i(z))$.

At the initial time ($t = 0$), the surface $z = H$ becomes free, and a depression wave propagates vertically downward along the magma. Throughout the process, the pressure in the magma chamber (on the boundary $z = 0$) is kept constant.

The process of volcanic eruption under consideration is described by the dynamics of a viscous fluid containing gas bubbles:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial z} = 0,$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g + \frac{1}{\rho} \frac{\partial}{\partial z} \left( \mu \frac{\partial v}{\partial z} \right).$$

Here $v$, $\rho$, and $p$ are the average velocity, density, and pressure of the medium, respectively, and $\mu$ is the viscosity of the magma melt.

Equations (2) must be supplemented by the equation of state

$$p = p_0 + \frac{\rho_0 c^2}{n} \left[ \left( \frac{\rho}{\rho_0 (1 - k)} \right)^n - 1 \right]$$

($c$ is the speed of sound in the fluid and $n$ is the constant in the Taite equation). In the derivation of Eq. (3), we used the Taite equation [16], in which the density of the fluid component is expressed in terms of its average density and volume concentration of the gas phase $k$ with the help of the equation of state for the two-phase mixture.

The dependence of the viscosity of the magma melt on the temperature $T$ and the mass concentration $C$ of the dissolved gas is given by the relation [2]

$$\mu = \mu^* \exp \left[ E_\mu (C) / (k_B T) \right],$$

(4)