1. INTRODUCTION

Improved measurement methods [1] and the possibility of fabricating metamaterials [2–4] and in particular, lefthanded materials (e.g., [5–8]), have motivated a number of recent investigations into dispersion forces on micro- and macro-objects with specially tailored magnetoelectric properties. In this context, dispersion forces on ground-state systems such as the Casimir–Polder (CP) interaction between a ground-state atom and a magnetoelectric body [9] and the van der Waals (vdW) interaction between two ground-state atoms in a magnetoelectric medium [10] have been studied. In both cases it was found lefthanded medium properties, being realized in certain finite frequency windows, are unable to noticeably affect these ground-state forces which depend on the medium response at all frequencies in an integral form. In the same spirit, the Casimir force between macroscopic bodies with metamaterial properties has been studied where possible anisotropy [11] and lefthandedness [12] has been taken into account. It could again be shown that the strength and the sign of the dispersion force is influenced by the strength of the magnetic properties rather than the lefthandedness [13, 14].

To enhance the impact of a lefthanded magnetoelectric response, it is natural to consider the resonant force components acting on excited systems, which depend on single selected frequencies. The vdW interaction has initially been studied for atoms in free space for the cases of one ground-state and one excited atom [15–17], two excited atoms [15] and three atoms with one of them being excited [18, 19]. The presence of ground-state media was taken into account in [20].

Similarly, the resonant CP interaction of a macroscopic ground-state body and an excited atom has been studied [21–23]. Applying the general results to a geometry involving a lefthanded slab, it was noted that the inclusion of material absorption is crucial when studying CP forces on excited atoms [24, 25]. Measurements of the CP energy of excited atoms are typically based on spectroscopic methods [26–28].

Excitations can also be present in the electromagnetic field, with thermal fields being an important special case. The impact of thermal fields on the Casimir force has been subject to discussions [29, 30] and experiments [31, 32]. While the Casimir force and the CP force [33–35] at thermal equilibrium are nonresonant, integral effects just like their zero-temperature counterparts, interesting phenomena can particularly arise for nonequilibrium systems. For instance, the CP force on an atom near a body held at a temperature different from that of the environment can be attractive or repulsive depending on the temperature difference [36], as has been confirmed experimentally [37, 38]. It has further been shown that even a ground-state atom can be subject to resonant force components when placed in a finite-temperature environment [39]. The vdW interaction between two atoms has recently even been studied in the presence of more general electromagnetic fields [40].

Although excitation has thus been included in the theories in various forms, dispersion forces on or in the presence of amplifying media have not been in focus yet, although such materials are indispensable in laser physics [41] and have recently attracted interest in the context of metamaterials [42, 43]. In particular, they are proposed to lead to repulsive forces [44] and may therefore be used to overcome the problem of stiction.
as has been pointed out by various authors [11, 46], the analysis leading to this prediction lacks a rigorous treatment of amplification. A microscopic approach to the problem was developed in [17] where the CP potential of a ground-state atom in front of an excited dilute gaseous medium, as well as the Casimir interaction between two dilute samples of excited gas atoms has been calculated. To go beyond such dilute-gas limits, an inclusion of amplification in the quantisation scheme is necessary. To our knowledge, the first approach to the problem was developed in [17] where the light propagates perpendicular to a dielectric amplifying slab. Within the framework of macroscopic QED, a full picture of the quantisation of the medium-assisted electromagnetic field in the presence of arbitrary (absorbing or amplifying) linear, causal media has been developed very recently [46]. This formalism is used in the present work to develop a consistent theory of the Casimir force on a body made of an amplifying metamaterial which generalises microscopic results beyond the dilute-gas limit.

The paper is organised as follows. In Section 2 we show how the quantisation scheme of medium-assisted electromagnetic field should be extended in the presence of body that is amplifying in a certain space- and frequency regime. After calculating the Casimir force on an arbitrary amplifying body in Section 3, we make contact to the microscopic CP forces on the excited atoms contained inside this body (Section 4). As an example, we consider the Casimir force on a dilute slab of excited gas atoms. The paper ends with the summary in Section 6.

2. MACROSCOPIC QUANTUM ELECTRODYNAMICS FOR AMPLIFYING MEDIA

We consider an arrangement of linear, local (isotropic) magnetoelectric bodies some of which are (linearly) amplifying in a limited frequency range and absorbing in the remaining frequency range. The bodies are described by spatially varying complex electric permittivity \( \varepsilon(\mathbf{r}, \omega) \) and magnetic permeability \( \mu(\mathbf{r}, \omega) \) that fulfil the Kramers-Kronig relations. The electric or magnetic response of the bodies is amplifying if \( \text{Im} \varepsilon(\mathbf{r}, \omega) = \varepsilon_f(\mathbf{r}, \omega) < 0 \) or \( \text{Im} \mu(\mathbf{r}, \omega) = \mu_f(\mathbf{r}, \omega) < 0 \) hold, respectively. Note that the strength of the amplification should be chosen such that the response to electromagnetic field is still linear. In particular we assume that the medium-assisted field is in an excited state where the medium is pumped in such a way that the state of the field can be regarded as quasi-stationary (for details, [46]).

The quantised electric field in the presence of the partially amplifying media can be given as the solution to the familiar Helmholtz equation

\[
\hat{\mathbf{E}}(\mathbf{r}, \omega) = \frac{1}{\mu(\mathbf{r}, \omega)} \nabla \times \left( \frac{\omega^2}{c^2} \mathbf{\varepsilon}(\mathbf{r}, \omega) \right) \nabla \times \hat{\mathbf{E}}(\mathbf{r}, \omega) = i \mu_0 \omega \hat{\mathbf{j}}_n(\mathbf{r}, \omega)
\]

according to

\[
\hat{\mathbf{E}}(\mathbf{r}, \omega) = i \mu_0 \omega \int d^3 r' G(\mathbf{r}, r', \omega) \cdot \hat{\mathbf{j}}_n(\mathbf{r}', \omega),
\]

where the classical Green tensor obeys the differential equation

\[
\left( \nabla \times \nabla \times - \frac{\omega^2}{c^2} \mathbf{\varepsilon}(\mathbf{r}, \omega) \right) \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \delta(\mathbf{r} - \mathbf{r}')
\]

together with the boundary condition at infinity. This differential equation can be cast into its equivalent form

\[
\left( \nabla \times \nabla \times - \frac{\omega^2}{c^2} \right) \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \delta(\mathbf{r} - \mathbf{r}') + \frac{\omega^2}{c^2} \left( \mathbf{\varepsilon}(\mathbf{r}, \omega) - 1 \right) \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega)
\]

\[
+ \nabla \times \left[ 1 - \frac{1}{\mu(\mathbf{r}, \omega)} \right] \nabla \times \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \delta(\mathbf{r} - \mathbf{r}') + i \mu_0 \omega \int d^3 s \mathbf{Q}(\mathbf{s}, \mathbf{r}, \omega) \cdot \mathbf{G}(\mathbf{s}, \mathbf{r}', \omega)
\]

by introducing the conductivity tensor [49, 50]

\[
\mathbf{Q}(\mathbf{r}, \mathbf{r}', \omega) = \sum_{\lambda = e, m} Q_\lambda(\mathbf{r}, \mathbf{r}', \omega),
\]

\[
Q_\lambda(\mathbf{r}, \mathbf{r}', \omega) = -i \varepsilon_\lambda \omega \left( \mathbf{\varepsilon}(\mathbf{r}, \omega) - 1 \right) \delta(\mathbf{r} - \mathbf{r}'),
\]

\[
Q_m(\mathbf{r}, \mathbf{r}', \omega) = -\frac{1}{i \omega \mu_0} \nabla \times \left[ 1 - \frac{1}{\mu(\mathbf{r}, \omega)} \right] \delta(\mathbf{r} - \mathbf{r}') \times \hat{\mathbf{E}}.
\]

It is well known that in the presence of amplification the roles of the noise creation and noise destruction operators are to be exchanged [51]. Hence, the noise current density reads

\[
\hat{\mathbf{j}}_n(\mathbf{r}, \omega) = \omega \left( \frac{\hbar}{\pi} \right) \left| \mathbf{\varepsilon}(\mathbf{r}, \omega) \right| \times \left[ \Theta[\varepsilon_f(\mathbf{r}, \omega)] \hat{\mathbf{f}}_e(\mathbf{r}, \omega) + \Theta[\varepsilon_f(\mathbf{r}, \omega)] \hat{\mathbf{f}}_e^\dagger(\mathbf{r}, \omega) \right]
\]

\[
+ \nabla \times \left( \frac{\hbar}{\pi \mu_0} \frac{\mu(\mathbf{r}, \omega)}{\left| \mathbf{\mu}(\mathbf{r}, \omega) \right|^2} \right) \times \left[ \Theta[\mu_f(\mathbf{r}, \omega)] \hat{\mathbf{f}}_m(\mathbf{r}, \omega) + \Theta[\mu_f(\mathbf{r}, \omega)] \hat{\mathbf{f}}_m^\dagger(\mathbf{r}, \omega) \right]
\]