Acoustooptic Interaction in Two-Dimensional Photonic Crystals: Efficiency of Bragg Diffraction

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Abstract—We obtain equations for the Bragg regime of acoustooptic diffraction of light in two-dimensional photonic crystals. We determine applicability conditions of the single-wave approximation, in which it is sufficient to take into account only one Fourier component of each of Bloch waves involved in the acoustooptic interaction. In the single-wave approximation, we obtain formulas that make it possible to estimate the acoustooptic figures of merit of a photonic crystal. We show that, in a photonic crystal, higher acoustooptic figures of merit can be achieved than in the materials that make up the crystal.

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INTRODUCTION

The diffraction of light by ultrasound in crystals is applied to create tunable spectral instruments, deflectors, and filters [1]. Because of the range of requirements on acoustooptic media, the search for new materials that have higher acoustooptic figures of merit in different ranges of the optical spectrum is an important problem. In this respect, it is interesting to study the prospects for the application of photonic crystals as acoustooptic materials. There are few publications in this field as of yet. In [2], the interaction of light with a surface acoustooptic wave in a photonic crystal is numerically simulated, using the modified transmission matrix method, which is traditional in the theory of photonic crystals [3]. The authors of [4] reported on the application of photonic-crystal fibers for creating a fiberoptic filter that can be tuned by the acoustic wave over a very wide spectral range.

In this work, we consider the problem on the acoustooptic interaction in a two-dimensional photonic crystal. In view of the above considerations, the modified transmission matrix method of [2] cannot be used to solve the posed problem because it is unreasonably laborious in solving two-dimensional problems. Attempts to rigorously pose the diffraction problem and, subsequently, to reduce it to numerical schemes seem to be even more cumbersome and, based on the results of calculations of this kind, it is difficult to make generalized conclusions.

As will be shown below, to describe the evolution of light wave amplitudes in a photonic crystal, one can obtain equations that, at a large length of the acoustooptic interaction (in the Bragg diffraction approximation), are slightly more complicated than similar equations for a homogeneous medium. In particular, the investigation of these equations makes it possible to determine the characteristics of the efficiency interactions between waves in photonic crystals.

INITIAL MODEL

Consider a photonic crystal that is a square lattice of cylindrical fibers of one material (e.g., fused quartz) in a matrix of the other material, i.e., silicon (Fig. 1). Unlike [2–4], in which collinear interaction geometry was studied, we will assume that light and sound can propagate at an arbitrary angle with respect to one another. Let an acoustic eigenwave propagate in a photonic crystal along the y axis and an electromagnetic eigenwave propagate at an angle \( \alpha \) to the x axis.

Due to the photoelastic effect, the light wave that propagates in the photonic crystal with a frequency \( \omega_i \) and wave vector \( k_i \) (incident) and the acoustic wave with a frequency \( \Omega \) and wave vector \( K \) are transformed
into a new light wave with a frequency $\omega_d$ and wave vector $k_d$ (diffracted). For the interaction between the optical and acoustic waves in the photonic crystal to be efficient, the following phase matching conditions should be satisfied:

$$\omega_i + \Omega = \omega_d, \quad k_i + K = k_d + G,$$  \hspace{1cm} (1)

where $G$ is the translation vector, which is a linear combination of the basis vectors $b_x, b_y$ of the reciprocal lattice of the photonic crystal,

$$G = mb_x + nb_y, \quad m, n = 0, \pm 1, \pm 2 \ldots$$  \hspace{1cm} (2)

The eigenwaves of the photonic crystal (the Bloch waves) are nonuniform, and their spectrum contains many spatial harmonics [5–9]. However, if phase matching conditions (1) are satisfied for fundamental harmonics, they are held for higher spatial harmonics as well.

The directions of the phase-matched propagation of waves upon acoustooptic interaction can be determined by the method of vector diagrams. The dispersion laws of optical and acoustic waves in photonic crystals are more complicated than in homogeneous media; therefore, the construction of the vector diagrams for them requires more laborious calculations [6]. As an example, Fig. 2 presents isofrequency curves for two light waves of the TE and TM polarizations that can propagate in the photonic crystal. Since the frequency of light is usually greater than the frequency of sound by many orders of magnitude, a change in the light frequency upon diffraction can be neglected, including $\omega_i = \omega_d$. As a consequence, the ends of the vectors $k_i$ and $k_d$ on the vector diagram lie on the same isofrequency curve.

By analogy with a homogeneous medium, we will term the diffraction the isotropic diffraction [1] (Fig. 2a) if condition (1) is satisfied for light waves of the same polarization. If the polarization of the diffracted wave differs from the polarization of the incident wave; i.e., condition (1) is satisfied for different branches of eigenwaves, this diffraction will be termed the anisotropic diffraction (Fig. 2b).

If the length of the phase-matched interaction in a photonic crystal is rather long, the Bragg diffraction regime will take place in it. As a criterion of the Bragg regime, similar to the case of the homogeneous medium, we can take the condition $Q \approx 1$, where $Q$ is the Klein–Cook parameter $Q = \frac{\lambda^2}{2\pi n v}$ [1]. Here, $l$ is the interaction length, $n$ is the refractive index of the medium, $\Omega$ is the cyclic frequency, and $v$ is the velocity of the sound. Therefore, for the Bragg diffraction to occur, the condition $l \gg \frac{2\pi n v^2}{\lambda^2 \Omega}$ should be satisfied. For example, if the frequency of the ultrasound is $\Omega = 2\pi \times 10^8$ rad/s (100 MHz) and the wavelength light is $\lambda = 1 \mu$m, the characteristic interaction length is $l \gg 300 \mu$m.

**EQUATIONS OF ACOUSTOOPTIC INTERACTION OF WAVES IN PHOTONIC CRYSTAL**

Based on the wave equation for the vector of the magnetic field $\mathbf{H}$ of a light wave, we obtain the follow-