INTRODUCTION

One of the most important problems of the modern statistical optics is related to the visualization of various optically nonuniform objects inside scattering media. In the last two decades, considerable interest in these problems is stimulated by medical and biological applications [1–3]. The main difficulty encountered upon viewing through biotissues is connected with multiple scattering, which leads to the loss of directionality of the incident light beam and, as a consequence, to the image blurring. Several approaches exist to the problem of viewing through biotissues in the visible and near IR ranges [1–3]. Almost all of these approaches are based on selecting photons (called snake photons) that propagate along nearly straight trajectories and carry information about optical inhomogeneities of the medium encountered on their way.

Among the approaches used for viewing through biotissues, due to its technical simplicity, an important place is occupied by the method that employs the polarization of light (see, e.g., [1, 3–19]). This method is based on the fact that the degree of polarization depends on the multiplicity of scattering and, correspondingly, on the photon path length in the medium. For this reason, two photons propagate along nearly straight trajectories and carry information about optical inhomogeneities of the medium encountered on their way.

The possibility of implementing the polarization method of imaging depends on the depolarizing properties of the medium, which are in turn affected by the concentration, size, and shape of scattering particles and their refractive index. A large number of experimental (see, e.g., [10, 15–19]) and theoretical [13, 20–27] papers have been devoted to studying the effect of parameters of the particles on polarization-related effects in the multiple-scattering media. The aim of these studies was to figure out regularities of the light depolarization in turbid media and to use this information for polarization viewing through biotissues.

Most theoretical studies devoted to multiple scattering of polarized light in turbid media consider methods of numerical calculations [11, 13, 20, 21, 23–25]. Simple analytical calculations that could explain the experimentally observed effects were absent until recently. The first results in this direction were obtained in [26–30] under simplified assumptions.

In this paper, we consider the light depolarization in optically turbid media of the biotissue type. We assume that the light scattering by large particles, in such media, occurs preferentially within small angles. This allows one to make use of the approximation of principal modes in the vector transfer equation [26–30] and to apply the small-angle Fokker–Planck approximation to describe each mode. In the proposed model, we consider the propagation of linearly polarized light in a medium with an inhomogeneity in the form of an absorbing half-plane. The degree of polarization and light intensity near the edge of the half-plane are found. The main attention is paid to analytical results describing the polarization state of the scattered light. The results thus obtained make it possible to explain the main observed effects and agree satisfactorily with the experimental data.
APPROXIMATION OF PRINCIPAL MODE
AND MODEL OF DEPOLARIZATION

Consider the normal incidence of a δ-pulsed light beam on the surface of a layer. To calculate the temporal profile of the degree of polarization of the pulse, one must solve the nonstationary (time-dependent) vector transfer equation. In the first approximation, we neglect the off-diagonal elements of the scattering matrix in the circular representation [26–30]. Then, the vector equation of radiation transfer breaks down into independent equations for the intensity and principal polarization modes [26–30].

The intensity obeys the scalar equation of the transfer

\[
\left( \frac{1}{c} \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial r} + \sigma_{\text{tot}} \right) I(r, \Omega, t) = \sigma \int d\Omega' a_t(\Omega, \Omega') I(r, \Omega', t),
\]

(1)

where \( \sigma_{\text{tot}} = \sigma + \kappa \) is the total extinction coefficient, \( \sigma \) and \( \kappa \) are the scattering and absorption coefficients of the medium, \( a_t(\Omega, \Omega') \) is the phase function (indicatrix) of scattering, and \( c \) is the speed of light in the medium. The unity vectors \( \Omega' \) and \( \Omega \) coincide with the directions of propagation of the light waves before and after the event of scattering; \( \Omega = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \); and \( \theta \) and \( \varphi \) are the polar and azimuthal angles, respectively.

For a broad beam, the boundary condition of Eq. (1) has the form

\[
I(z = 0, \Omega, t) = I_0 \delta(t) \delta(\Omega - \Omega_0),
\]

where \( \Omega_0 \) is the inner normal to the surface.

For photons that propagate close to straight lines, we may use the small-angle approximation [31]. In this approximation, Eq. (1) has the form [32–34]

\[
\left( \frac{\partial}{\partial z} + \frac{\theta^2}{2} \frac{\partial}{\partial \Delta} \right) \tilde{I}(z, \theta, \Delta) = \tilde{I}_a,
\]

(2)

where \( \tilde{I}(z, \theta, \Delta) = c^{-1} \exp(\kappa c t) I(z, \theta, \Delta), \Delta = ct - z \) is the difference between the path length \( ct \) and the depth \( z \).

Below, we will use the collision integral \( \tilde{I}_a \) in the Fokker–Planck approximation [32–34]

\[
\tilde{I}_a = \frac{\sigma_t}{\theta^2} \frac{\partial}{\partial \theta} \tilde{I}_a,
\]

(3)

where \( \sigma_t = \sigma(1 - \langle \cos \gamma \rangle) \) is the transport scattering coefficient, and \( \langle \cos \gamma \rangle \) is the mean cosine of the angle of single scattering. This model takes into account the strongly anisotropic light scattering in phantoms of the biotissues, e.g., in aqueous suspensions of latex particles, and in biological media [35, 36].

The small-angle approximation is justified when the mean square of the multiple scattering angle \( \langle \theta^2 \rangle_{z,t} \) at the depth \( z \) and the moment \( t \) meets the inequality \( \langle \theta^2 \rangle_{z,t} \ll 1 \) [33, 34]. According to [33], Eqs. (2) and (3) admit the exact analytical solution, which shows that \( \langle \theta^2 \rangle_{z,t} = 4 \Delta/z \). For this reason, the small-angle approximation is applicable when \( \Delta = ct - z \ll z \).

The polarization state of a linearly polarized incident beam is described by the transfer equation for the principal mode of the linear polarization [26–39]

\[
\left( \frac{1}{c} \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial r} + \sigma_{\text{tot}} \right) W(r, \Omega, t) = \sigma \int d\Omega' a_t(\Omega, \Omega') \exp(2i(\chi_+ - \psi)) W(r, \Omega', t),
\]

(4)

where \( a_t(\Omega, \Omega') \) is the first diagonal element of the scattering matrix in the circular representation [26, 27, 37], and the angles \( \chi_+ \) and \( \psi \) are defined by the equality

\[
\chi_+ = \pi - (\beta + \beta'), \quad \psi = \varphi - \varphi',
\]

\[
\cos 2\beta = 1 - 2 \sin^2 \theta' (1 - \cos^2 \psi),
\]

\[
\sin 2\beta = 2 \sin \theta' \cos \theta' \sin \theta - \cos \theta \sin \theta' \cos \psi \sin \psi, \quad 1 - (\Omega \Omega')^2
\]

\[
\Omega' = \cos \theta' \cos \varphi + \sin \theta \sin \varphi \cos \psi.
\]

The functions \( \cos 2\beta' \) and \( \sin 2\beta' \) differ from the functions \( \cos 2\beta \) and \( \sin 2\beta \) by the replacement of \( \varphi \) with \( \varphi' \) \((\theta, \theta', \varphi, \varphi') \) are the polar and azimuthal angles of the vectors \( \Omega \) and \( \Omega' \), respectively.

For spherical particles, we have

\[
a_t = \frac{1}{2} (|A||A| + |A|), \quad a_+ = \frac{1}{4} (|A||A| + |A|),
\]

where \( A \) and \( A_\perp \) are the scattering amplitudes of the waves polarized in the plane of scattering and normal to it. The amplitudes \( A_\parallel \) and \( A_\perp \) are given by the Mie formulas [31, 37, 38].

The boundary condition for Eq. (4) has the form

\[
W(z = 0, \Omega, t) = W_0 \delta(t) \delta(\Omega - \Omega_0).
\]

For the totally polarized incident light, \( W_0 = I_0 \).

As applied to Eq. (4), the small-angle approximation is derived as follows. By expanding the angle-dependent coefficients in the right- and left-hand sides of Eq. (4) in terms of small angle \( \theta \) and retaining only the first nonvanishing terms, we can reduce Eq. (4) to the form

\[
\left( \frac{\partial}{\partial z} + \frac{\theta^2}{2} \frac{\partial}{\partial \Delta} \right) \tilde{W}(z, \theta, \Delta) = \tilde{W}_a,
\]

(5)