INTRODUCTION

Perfecting information systems to heighten their speed and compactness, with allowance for ecological problems of saving energy and resources today makes disperse media convenient objects for constructing miniature devices based on solitary dielectric microparticles and microfibers. This is related to the fact that weakly absorbing dielectric microparticles act as optical resonators and, as a consequence, in these cavities, morphological resonances or high-\(Q\) whispering gallery modes arise. The microcavities based on dielectric microparticles of different symmetry (spherical, cylindrical, ellipsoidal, etc.) have additional advantages, among which are considerable reduction of thresholds for different nonlinear phenomena, strong focusing of the field, low sensitivity to damage and overheating, and convenience of matching microspheres and microcylinders to optical fibers due to the scaling effect.

One of the first studies of eigenmodes in a dye-impregnated cylindrical microlaser and their use for measuring parameters of microcavities was performed in [1, 2]. Microfibers provided nearly lossless excitation of high-\(Q\) whispering-gallery modes in optical cavities, which were extremely sensitive to variations of the environment and could be used as sensors [3, 4]. Lasing on a ring microcavity 2 mm in diameter formed by dye-doped polymeric optical microfibers 3.8 \(\mu\)m in diameter was observed in [4]. Optical processes in cylindrical microcavities have been the subject of extensive experimental research [5–8]. These papers, however, were of essentially demonstrative character aimed at illustrating the potential of such systems. At the same time, the observed optical phenomena remain not quite clear and need to be theoretically grounded. In particular, rather many specific features of the effect of spatial structure of the electromagnetic field upon the process of low-threshold lasing remain unexplored, hence the corresponding theoretical research.

THEORETICAL MODEL OF THE TRANSFORMATION OF AN ELECTROMAGNETIC FIELD IN A SEMICONDUCTING MICROCYLINDER

Microparticles of cylindrical symmetry play the role of specific optical microcavities with a certain set of eigenmodes. Each eigenmode is characterized by a certain eigenfrequency and \(Q\)-value depending both on the diffraction parameter of the microcylinder (\(\rho = \pi \lambda / 2R\), \(R\) is the microcylinder radius, and \(\lambda\) is the laser light wavelength) and on the values of the optical constants of the particle material. An important electrodynamic characteristic of the microcavity, along with its \(Q\)-value, is the spatial structure of its internal electromagnetic field, which is a result of summations of spatial structures of the eigenmode fields. Knowledge of the field distribution inside the particle is also highly important for further studying of the characteristics of transformation of light by complex objects such as a semiconductor microlaser. The resonant eigenmodes are usually realized in media with an extremely small absorption, i.e., in media that are nearly transparent.

The spatial structure of the field in each eigenmode is characterized by the number \(n\) that determines the amount of peaks in the intensity distribution of the internal field upon variation of the polar angle \(\varphi\) from 0° to 180° and by the order \(s\) equal to the number of peaks in the intensity distribution of the eigenmode field over the radius \(r\). The external finesse of the morphological resonances can be rather high and increase with increasing mode number, while the growth of the
mode number, on the contrary, leads to an increase of the external quality factor for the same number. Analysis of the experimental data shows that the growing external quality factor is not always accompanied by increasing field intensity inside the microparticle.

Consider microparticles of cylindrical symmetry. The field inside cylindrical microparticles can be presented as a superposition of the electric (or transverse-magnetic, TM) and magnetic (or transverse electric, TE) waves. The expression for the spatial distribution of the energy density of the resonant mode looks as follows:

\[ \frac{I}{I_0} = \left( |E_x|^2 + |E_y|^2 + |E_z|^2 \right) \cdot \frac{E_0^2}{\epsilon_0}. \] (1)

Here, \( E_x, E_y, \) and \( E_z \) are the electric field components inside the microcylinder; \( E_0 \) is the electric field of the incident laser beam; \( I_0 \) is the intensity of the incident laser beam; and \( I \) is the spatial distribution of the energy density inside the cylindrical microparticle. Components of the electric field at a given point inside the homogeneous circular microcylinder can be obtained from the theory of diffraction of electromagnetic radiation by a cylindrical particle \([9, 10]\). These components of the electric field at a given point inside the cylinder irradiated by a plane electromagnetic wave along the direction normal to its axis, for the case of the TM polarization, can be written as

\[ E^{(1)}_r = E^{(1)}_\varphi = 0, \]
\[ E^{(1)}_z = i m E_0 \]
\[ \times \sum_{l=1}^{\infty} (-i)^l \cos(l\varphi) J_l(mkr) d_l + J_0(mkr) d_0. \] (3)

The expressions for the electric field components inside the cylinder, for the TE polarization, have the form

\[ E^{(2)}_x = 0, \]
\[ E^{(2)}_r = -\frac{2E_0}{mkr} \sum_{l=1}^{\infty} (-i)^l \sin(l\varphi) J_l(mkr) c_l, \]
\[ E^{(2)}_\varphi = -m E_0 \]
\[ \times \sum_{l=1}^{\infty} (-i)^l \cos(l\varphi) J'_l(mkr) c_l + J'_0(mkr) c_0. \] (6)

In Eqs. (3)–(6), \( m = n - i\kappa \) is the complex refractive index of the particle material; \( n \) is the real part of the refractive index; \( \kappa \) is its imaginary part; \( z, r, \) and \( \varphi \) are the coordinates of the point inside the particle in the cylindrical coordinate system; \( k = 2\pi/\lambda; \) \( \lambda \) is the laser light wavelength; \( J_l \) is the first-kind cylindrical Bessel function; and \( J'_l \) is the derivative of the Bessel function. The coefficients of expansion for the internal field \( d_l \) and \( c_l \) in Eqs. (3)–(6) have the following form:

\[ d_l = -\frac{J_l(\rho)}{mJ_l(mp)}(i + b_l), \] (7)
\[ c_l = -\frac{J_l(\rho)}{mJ_l(mp)}(i + a_l), \] (8)
\[ b_l = -i \frac{D_l(\rho) - mD_l(mp)}{mG_l(\rho) - D_l(mp)}, \] (9)
\[ a_l = -i \frac{mD_l(\rho) - D_l(mp)}{mG_l(\rho) - D_l(mp)}. \] (10)

Here, \( \rho = 2\pi R/\lambda \) is the diffraction parameter of the particle and \( R \) is the radius of the cylinder.

The resonant structure of the internal field of the circular cylindrical microparticle varies as \( \cos(l\varphi) \) providing 2\( l \) maxima in the distribution of the internal intensity, since the angle \( \varphi \) varies from 0 to \( 2\pi \) in the plane normal to the axis of the cylinder. Since the resonant structure of the internal field for the circular cylindrical microparticle is observed only at normal incidence of the light upon its surface, we do not consider other angles of incidence of the laser light. From Eqs. (1)–(10), we see that the energy density in the spatial structure of the resonant mode for the circular cylindrical particle can be characterized by squares of modules of the amplitude coefficients of the internal field \( |d_l|^2 \) (TM polarization) or \( |c_l|^2 \) (TE polarization). Therefore, the changes in the amplitude coefficients upon variation of the diffraction parameters of the microparticle or refraction and absorption indexes of its material will determine the character of spatial distribution of the internal field in the microcylinder and will make it possible to evaluate \( Q \)-values of the microcavity modes and field distribution of its spatial structures, i.e., inhomogeneities of the electric field distribution in the microcylinder.

**DISCUSSION OF THE TRANSFORMATION OF AN ELECTRIC FIELD IN A SEMICONDUCTING MICROCYLINDER**

The semiconductor laser was modeled by a homogeneous circular microcylinder. The optical constants of the microcylinder material were taken from \([11]\). For the case \( \lambda = 1.5–1.6 \mu m \), for a semiconductor heterostructures, \( m \equiv 3.35–10^{-8} \).

Theoretical model for the transformation of the electromagnetic field by the semiconductor microcylinder used here makes it possible to determine the distribution of the fields of its spatial structures, i.e., the inhomogeneity in the distribution of optical fields in the microparticle. The spatial distribution of the optical fields inside homogeneous microcylinders, under resonant conditions, is strongly inhomogeneous, all