An Increase in the Sensitivity of the Saturated Absorption Method in the Multimode Regime

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Received July 12, 2012

Abstract—A possibility for an increase in the signal-to-noise ratio in the laser spectroscopy that is free of Doppler broadening and is based on the saturated absorption is considered. The application of the counterpropagating laser beams in the multimode regime is proposed. The number of atoms that effectively interact with the field, and, hence, the intensity of a narrow resonance in the line shape can be increased due to the interaction of the counterpropagating modes with different frequencies. It is demonstrated that, for the intrinsic photon noise, the signal-to-noise ratio can be increased by a factor of \( \sqrt{N} \), where \( N \) is the number of modes. For the remaining noises (fluctuations of the radiation power, noise of photodetector, etc.), the signal-to-noise ratio may increase by a factor of \( N \).

DOI: 10.1134/S0030400X13030041

1. INTRODUCTION

One of the main methods of laser spectroscopy that is free of Doppler broadening employs the saturated absorption [1, 2]. The method is based on the selection of atoms with respect to velocities under the resonance interaction of the optical field and gas. This effect makes it possible to obtain relatively narrow resonances at the center of a line, such that the widths are significantly less than the Doppler width. The method is widely employed for the stabilization of laser frequency and precision laser spectroscopy. In particular, note the measurements of the transition frequencies for the fine structure of the \( ^2P \) level in helium [3], which allowed refinement of fine-structure constant \( \alpha \). The resonance of saturated absorption is used as the reference signal for the stabilization of laser frequency. An optical frequency standard with ultracold calcium atoms [4] exhibits a relative uncertainty of frequency of about \( 10^{-15} \).

Normally, the saturated absorption method is related to the absorption power resonance in the presence of a standing wave (saturated absorption resonance). However, the effect is also possible upon the interaction of counterpropagating waves with different frequencies (see, for example, [5]). This circumstance makes it possible to increase the number of gas atoms that effectively interact with the field and, hence, the intensity of the narrow resonance in the line shape. The method is not widely used, in spite of the fact that an increase in the signal-to-noise ratio was experimentally demonstrated in [6]. However, note the progress that has been made in experimental techniques and the advent of new laser sources with self-mode-locking (femtosecond lasers and laser diodes). We assume that the method may provide a significant increase in the signal-to-noise ratio, which can be employed in the modern frequency standards for an increase in the accuracy and stability.

In this work, we analyze a possibility for an increase in the signal-to-noise ratio with the aid of the resonance of saturated absorption in the multimode regime of the counterpropagating waves. We consider a scheme for the measurement of small oscillations of surface using the saturated absorption resonance. It is demonstrated that, for the intrinsic photon noise, the signal-to-noise ratio can be increased by a factor of \( \sqrt{N} \), where \( N \) is the number of modes. For the remaining noises (fluctuations of the radiation power, noise of photodetector, etc.), the signal-to-noise ratio may increase by a factor of \( N \).

2. MEASUREMENT OF SMALL PERIODIC DISPLACEMENTS USING NARROW OPTICAL RESONANCES

The measurement method is based on the detection of minor variations in the laser frequency that result from variations in the cavity length or refractive index of the medium in the presence of perturbations [7]. Figure 1 demonstrates the measurement principle.

A Fabry–Perot cavity contains an amplifying medium and a gas cell that makes it possible to obtain the saturated absorption resonance the width of which is significantly less than the Doppler width. A periodic perturbation at frequency \( \Omega \) acts upon one mirror of
the laser cavity and causes variations in cavity length \( L \) by \( \Delta L \), so that the laser frequency is changed:

\[
\omega(t) = \omega + \Delta \omega \cos(\Omega t),
\]

Here, \( \Delta \omega = \omega \Delta L / L \) and \( \omega \) is the laser frequency. Using a narrow resonance in the absorption line shape, we transform a variation in the laser frequency into variation in laser power \( \Delta P \) that represents a signal and is measured using a photodetector.

For the observation of the resonance, the signal must be detected in the presence of noise. Note the existence of the intrinsic photon noise, which is related to the quantum fluctuations of the laser radiation. For the estimation of the noise, we assume that laser power \( P_0 \) is measured by the detector in frequency band \( \delta \omega \) and the detector measures the laser radiation as pulses with duration \( \tau = 1/\delta \omega \). For the Poisson distribution function of photons, the mean number of photons per pulse is

\[
n = \frac{P_0 \tau}{\hbar \omega},
\]

and the variance is

\[
\Delta n = \sqrt{n} = \sqrt{\frac{P_0 \tau}{\hbar \omega}}.
\]

Using photon energy \( \hbar \omega \), we represent the noise power in frequency band \( \delta \omega \) as \( P_{\text{noise}} = \hbar \omega \Delta n / \tau \). Thus, we have

\[
P_{\text{noise}} = \sqrt{\hbar \omega P_0 / \tau}.
\]

The ratio of the signal power to the noise power is given by

\[
\frac{P_{\text{signal}}}{P_{\text{noise}}} = \frac{\Delta P}{\sqrt{\hbar \omega P_0 / \tau}}.
\]

The maximum sensitivity is reached when the laser frequency is tuned to the range of the maximum slope of the resonance curve. The sensitivity of the transformation increases with an increase in the resonance intensity and a decrease in the resonance width.

Below, we consider the multimode lasing. In this case, it is expedient to represent the field as a set of counterpropagating waves with different frequencies that are symmetric relative to the saturated absorption resonance.

### 3. SATURATED ABSORPTION RESONANCE FOR TWO COUNTERPROPAGATING WAVES

We consider a gas that interacts with two counterpropagating waves the frequencies of which are symmetric relative to transition frequency \( \omega_{21} \) between levels 2 and 1. We assume that waves with frequencies \( \omega^+ \) and \( \omega^- \) propagate along the \( z \) axis and in the opposite direction, respectively, and frequency \( \omega_{21} \) of the atomic transition is close to frequencies \( \omega^+ \) and \( \omega^- \). The projection of the atomic velocity in the beam along the \( z \) axis is \( \nu \). The wave with frequency \( \omega^+ \) interacts with atoms the velocities of which are found from the resonance condition \( \omega^+ = \omega_{21} - kv \), where \( k \) is the wave vector. For the oppositely directed wave, the resonance condition is written as \( \omega^- = \omega_{21} + kv \). In general, the waves interact with different atoms. When the condition

\[
\frac{\omega^+ + \omega^-}{2} = \omega_{21}
\]

is satisfied, the waves interact with the same atoms (Fig. 2), the velocities of which are represented as

\[
\nu = \frac{\omega^+ - \omega^-}{2k}.
\]

This circumstance leads to resonance in the absorbed power of one wave when \( \omega^+ + \omega^- = 2\omega_{21} \). When the Doppler line width is significantly greater than the homogeneous line width and the intermode distance, the expression for the saturated absorption resonance is written as

\[
P(\omega) = P \left( 1 - \frac{\kappa(\Gamma/2)^2}{(\omega - \omega_{21})^2 + (\Gamma/2)^2} \right),
\]

where

\[
\kappa = \frac{\Gamma}{2 \omega_{21} - \nu}.
\]