Remote Method of Determining the Coordinates of Points on a Planetary Surface

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Abstract — A method and an algorithm for determining the coordinates of points on the planetary surface are described. The coordinates are determined using photographs. To solve the problem, the spacecraft coordinates need to be determined at five trajectory points. The spacecraft trajectory is considered to be a plane. The method is applicable for determining the coordinates of points on the Earth’s surface and on the surface of other planets.

Keywords: remote sensing, photography, coordinates of points on the planetary surface

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INTRODUCTION

To study planets, we have to introduce planetocentric and planetographic coordinates (www.solar-model.ru, 2013). For a number of planets, specific names of coordinates are used, for example: hermographic coordinates for Mercury (Hermes), venusian coordinates for Venus (Tyuflin, 1986), geographic coordinates for Earth, selenographic coordinates for Moon, areographic coordinates for Mars (Ares), etc. Generally, however, we can talk about planetocentric and planetographic coordinates when we deal with unknown planets or with a technique of determining the coordinates irrespective of specific planetary features.

In practice, we can encounter a situation when researching the planetary surface when it is necessary to determine the coordinates of surface points using photographs taken by a spacecraft (SC) (Tyuflin, 1986; Manned Mission to Mars, 2006; Savinykh and Tsvetkov, 2012). In this method, there are no reference points on the surface with known coordinates. This rules out the use of classical methods of processing the images in photogrammetry based on the presence of these points, which is considered as a necessary condition (Tyuflin, 1986; Bugaevskii and Portnov, 1984; Urmaev, 1989). A number of techniques (Kashkin and Sukhinin, 2001) are oriented toward digital methods and do not resolve questions of direct intersection. This paper is devoted to the method of determining the point coordinates on the surface of any planet when reference points are not available on this surface.

1. CONDITIONS OF APPLYING THE METHOD

The general principles of the problem solution are given in (Tsvetkov, 2011). The conditions needed for the applicability of the method are as follows. A spacecraft (SC) that has an on-board camera for photographing the surface and an inertial device approaches the planetary surface. The camera is rigidly fixed in the SC body. The inertial device determines a relative location of the SC consecutively at five points in its trajectory. The gravitational anomalies are assumed to be absent, suggesting that the SC trajectory can be considered as a second-order curve. It is not important whether the SC trajectory is elliptical, parabolic, or hyperbolic. A problem using the general equation of a second-order curve comprising all the listed variants is solved. This simplifies the search for a solution. In principle, there can be several versions of models for calculating the coordinates. Our model of the coordinate determination is the simplest one and includes the following stages:

1. The determination of the coordinates of SC trajectory points.

2. The analytical determination of the SC trajectory and the normals to the trajectory at the photography points.

3. The emulation of the geodetic measurements.

4. The determination of planetographic coordinates based on the solution of a direct intersection under the condition of scale equality.
1.1. Determination of the Coordinates of the SC Trajectory Points

The spacecraft is equipped with an inertial sensor. This allows us to fix its relative location at specified moments in time, providing a possibility to measure the coordinates of points in order to determine the trajectory. Five points \((T_1, T_2, T_3, T_4, T_5)\) are required for a second-order curve. Two more trajectory points \((T_6 \text{ and } T_7)\) are measured at the photography points \(S_1\) and \(S_2\) (the shooting points). The two latter measurements define a basis for photography.

1.2. Analytical Determination of the SC Trajectory and a Normal to the Trajectory at a Shooting Point

According to the equations of motion, an SC trajectory should be a second-order curve. If there are no perturbing influences, the SC moves along a trajectory that is a segment of a plane curve in space. This provides a basis for solving the problem of determining the trajectory parameters in a plane, i.e., to operate only with the trajectory coordinates \(x\) and \(y\) in the plane of motion:

\[ ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0. \]

Equation (1) is linearized and is linearly solved relative to the coefficients \((a, b, c, d, \text{ and } e)\) provided that coordinates of five points are available.

The equation of a tangent to a second-order curve has the form

\[ ax + by + d = 0 \]

or

\[ y = -ax/b - d/b. \]

Using (2), it is not difficult to determine the equation of a normal to the second-order curve at the point with the coordinates \((x, y)\)

\[ y = bx/a + h, \]

where \(h\) is the constant that is determined for the known coordinates of the shooting point \(S\).

The substitution of the shooting point coordinates into (3) yields a value of the normal to the trajectory at this point.

1.3. Determination of the Planetographic Coordinates

The shooting camera is rigidly fixed on the SC and does not change its location during the flight with respect to the intrinsic coordinate system of the SC. The location of the camera is arbitrary relative to the trajectory but is fixed. For example, the optical axis can form an angle to the trajectory plane or lie in this plane. Therefore, the camera positions at points \(T_6\) and \(T_7\) only differ in the convergent angle and the photography basis. Two other angles do not vary.

The coordinates of the photograph center and the focal length of the camera are known. These parameters are used in calculations.

The calculation algorithm comprises the following stages:

1. The coordinates of the SC trajectory points are determined using the inertial sensor in order to determine the coefficients in Eq. (1).

2. The coordinates of the shooting points \(S_1\) and \(S_2\) of the SC trajectory are determined using the inertial sensor.

3. The equations of normals for these points are derived from Eq. (3) using the coordinates of points \(S_1\) and \(S_2\).

4. The convergent angle between the directions of the optical axes (normals) of the photographs at points \(S_1\) and \(S_2\) is determined using the directions of the normals.

5. Using a pair of photographs taken at points \(S_1\) and \(S_2\), the corresponding points on the surface of the heavenly body are visually identified.

6. The coordinate system of the photograph taken at point \(S_1\) is assumed to be a reference coordinate system.

7. The coordinate system of the photograph taken at point \(S_2\) is tilted with respect to the reference coordinate system at an angle that is determined in stage (4).

8. The corresponding points associated with a common point on the planetary surface are identified in the shooting images (Lyskov and Tsvetkov, 1976).

9. The vectors \(r(u, p, f)\) are constructed to corresponding points of a pair of photographs. The vector components are the coordinates of the photograph points \((u, p)\) and the focal length of the camera \(f\).

10. The transformation from coordinate photogrammetric measurements to angular geodetic ones is made (Tsvetkov, 1986) using the division of the coordinates by the focal length.

11. The problem of direct intersection is solved under the condition of scale equality (Tsvetkov and Kheblenikova, 1987).

Figure 1 shows a model diagram of constructing the coordinates of point \(M\).

The designations in Fig. 1 are as follows: \(T\) is a segment of the plane trajectory in which the shooting points lie; \(S_1\) and \(S_2\) are the shooting points; \(B\) is the base of the photography; \(M\) is the point for which the planetographic coordinates need to be determined; \(R_1\) and \(R_2\) are the position vectors that do not generally lie in the same plane (i.e., it is admitted that the condition of coplanarity can be broken while this condition is considered to be necessary in classical photogrammetry). These vectors can be calculated and may be called intersection vectors. \(D\) is the non-coplanarity vector, a segment that links the ends of the position vectors and comprises the point \(M\) to be determined. If the position vectors lie in the same plane (the coplanar vectors), this segment is zero.

Tentatively, the unit position vectors \(r_1\), \(r_2\), and \(d\) are collinear with the vectors \(R_1\), \(R_2\), and \(D\) but are determined using the photograph coordinates. The \(r_1\)