Review and Assessment of the Theories of Stable Alluvial Channel Design

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Abstract—Deformable alluvial channels are known to adjust their geometry and slope to achieve stable conditions for a specified influx of water and sediment. Designing the stable alluvial channel has been a captivating topic for scientists and engineers around the globe for years. The work which was commenced by Kennedy in 1895 has been continued and various approaches have been given so far, some of which are quite interrelated and others emerged with different ideas. In this comprehensive study, some of the classic and widely accepted approaches published in the literature have been thoroughly reviewed and have been verified with available river regime data. The data set has been sub divided into three categories based on the median bed material size (sand, gravel and cobble or boulder), in order to examine the applicability of various methods available for the design of stable alluvial channels. Detailed discussion related to the properties of the intercept coefficients in power function theory is not available in published literature. In this study, the coefficients are first calibrated and then applied with the respective exponents in order to derive the hydraulic geometry. Further, the derived hydraulic geometry from various approaches is summarized and discussed with comparative view point. The analysis shows that prediction from recently developed model based on the principle of maximum entropy and minimum energy dissipation is better than other approaches for the entire range of data set. The same model has been further generalized by assuming a wide trapezoidal channel cross-section through which an improvement in the prediction has been observed.

Keywords: alluvial channels, analytical approaches, extremal hypotheses, hydraulic geometry, regime concept, principle of maximum entropy and minimum energy dissipation rate

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Alluvial channels are the man-made channels constructed in alluvium which embodies deformable bed and banks and mostly used for irrigational and navigational purposes. The equilibrium geometry of alluvial channels remained a topic of fundamental scientific and engineering interest for years. Cross-sectional geometry and slope of alluvial channels change with the variation in the sediment load transported by them. The phenomenon of sedimentation and erosion has been observed in alluvial channels when heavily silt-laden water flows through them. A large amount of expenditure is incurred on maintenance of channels in desilting and on the other hand scouring of the bed, hinders the capacity of the channel.

An alluvial channel typically has three degrees of freedom i.e. width, depth and slope. In order to determine channel geometry completely, a set of three independent equations are thus required. These three equations can be obtained either empirically or analytically [1]. In empirical approach, the required equations are obtained by analyzing the available data from stable alluvial channels with the laws of statistics.

This empirical approach of attaining a set of three independent equations by analyzing the available data from existing stable channels is frequently known as the regime theory and was originated by the engineers working on irrigation canals of India and Pakistan.

In the analytical approach on the other hand, the required equations are obtained theoretically and according to White et al. [2] “The analytical methods rely on specifying equations which describe the dominant individual processes such as sediment transport, flow resistance and bank stability. These approaches can only be successful if the dominant processes are correctly identified and appropriate equations exist to describe them adequately.” The objective of this study is to review various approaches available in the literature for designing the stable alluvial channel and their pertinence has been accentuated. Further, a generalized model for the prediction of hydraulic geometry of stable alluvial channel has been tried to develop, which is the extension of the model proposed by Singh et al. [3, 4].

1 The article is published in the original.
EMPIRICAL APPROACHES

Empirical approaches are those in which the set of three equations is sought by analyzing the available data from the existing stable alluvial channels, with the laws of statistics in order to derive the hydraulic geometry.

The Regime Concept

In regime methods, the stable channel is designated as 'regime channel.' One common observation that alluvial channel adjusts its dimensions in response to the variations in the size and quantity of sediments carried by it, lays the foundation of regime concept. Regime concept has been started by Kennedy [5] and further strengthened by Lindley [6] and Lacey [7]. Kennedy [5] stated that a channel is said to be in a state of 'regime' if there is neither silting nor scouring in the channel. Lindley [6] proposed equations for depth and channel width recognizing that the bed and the banks as well are subjected to scour or fill, which were:

\[ U_{cr} = 0.95D^{0.57} \text{ and } U_{cr} = 0.57B^{0.355}, \]  

where \( U_{cr} \) is the critical velocity at which for a given depth \( D \), silting is just prevented i.e. maintaining the entire sediment load without eroding the channel and \( B \) is the average width between banks of channel. Lacey’s regime method was published from 1930 to 1958 in a series of papers. It has been stated by Lacey that when water flows through an excavated channel, the silt carried by the water may get deposited in the upper reaches. Lacey’s method should be applied when bed-material size is in the range of 0.15 to 0.40 mm and discharge between 5 and 5000 cfs. Blench [8] extended the previous regime methods to include the bank material characteristics [9]. Simons and Albertson [10] studied the canals of India–Pakistan and USA and classified them into five types on the basis of the different geotechnical channel conditions. The range of discharges studied was 5 to more than 9000 cusecs, with an average sediment discharge of 156 to 8000 ppm. The mean size of the bed material varied from 0.1 mm to 7.5 mm.

Power Function Theory

Leopold and Maddock [11] developed a set of empirical equations to express the variation of water surface width, mean depth and velocity for a channel in form of power functions of discharge for both at a-station and downstream the channel. The relations are:

\[ B = aQ^b, \quad D = cQ^d \text{ and } U = kQ^m, \]  

where \( B \) is channel width, \( D \) is mean flow depth, \( U \) is velocity, \( Q \) is the discharge, \( b, f \) and \( m \) are the exponents and represent the slope of the three lines on the graph of variation of width, depth and velocity with mean annual discharge, respectively. \( a, c \) and \( k \) are the constants and are respectively, the values of width, depth and velocity when the discharge is unity. The average values of \( b, f \) and \( m \) for 20 river cross-sections of United States found by Leopold and Maddock [11] were 0.5, 0.4 and 0.1 respectively. The work of Leopold and Maddock [11] shows that discharge, width and depth increase in a similar manner for most of the streams in the downstream direction due to increase in drainage area but slight increase is observed in the velocity because of decreasing slope downstream. Singh [12] has compiled the average values of exponents \( b, f \) and \( m \) for the downstream hydraulic geometry.

ANALYTICAL APPROACHES

Analytical approaches constitute the derivation of hydraulic geometry of stable alluvial channels, from theoretical considerations of laws governing fluvial hydraulics and sediment transport under the dynamic equilibrium. Some of the analytical approaches are: tractive force theory [13, 14], theory based on conservation of mass and momentum [15] and echelon matrix procedure developed by Martin [16]. Lane [14] defined the stable channel as an unlined earth channel which carries water, bed and banks of which are not scoured objectionably by moving water through it and at the same time there in no objectionable deposition of sediments. Smith [15] derived the form parameters of width, depth, velocity and slope for the downstream changes in the channel geometry from three necessary conditions for the existence of steady state, finite width channel as a surface of \( z = z(x, y, t) \); (1) sediment mass is conserved during transport, (2) the channel has the form just sufficient to carry its total water discharge and (3) the channel has the form just sufficient to carry its total sediment discharge. By applying the continuity equation for water discharge, Manning’s equation for the magnitude of water flux in the downstream direction and conservation principle for the flux of sediment mass, Smith [15] derived the following mathematical expression for the channel model:

\[
\frac{\partial z}{\partial t} = \frac{\partial}{\partial x} \left( k_1 k_2 D^{5n/3} S_{w}^{m+n/2} \right) - \frac{\partial}{\partial y} \left( k_1 k_2 D^{5n/3} S_{w}^{m+n/2} \right) \frac{\partial D}{\partial y},
\]

\[
\int_{y_1}^{y_2} k_1 D^{5/3} S_{w}^{1/2} dy = Ax,
\]

\[
\int_{y_1}^{y_2} k_1 k_2 D^{5n/3} S_{w}^{m+n/2} dy = Lx,
\]

where \( z \) is the elevation of the channel bed, \( x \) and \( y \) are the downstream and transverse directions respectively, \( K_1, K_2, A \) and \( L \) are the constants, \( S_{w} \) is the water surface slope, \( D \) is the depth and \( m, n \) usually take the val-