The Higgs Field and Extra Dimensions

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Abstract—We discuss the possible geometric origin of the Higgs field of the Standard model. A model is built where the Higgs field, along with the gauge fields, appears from off-diagonal components of the multidimensional metric. In the low-energy limit, the Higgs field potential and its interaction with the gauge fields of the electroweak theory are reproduced.

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1. INTRODUCTION

Although the Standard model (SM) of particle physics is a basis of modern physics, it has a number of shortcomings, and, in particular, its very origin is still remaining problematic. Why does the theory possess this particular symmetry \( SU(2) \otimes U(1) \) in the present case? What is the origin of the Higgs field, gauge fields and matter fields? These are only some of the questions yet to be answered.

In this paper, we would like to discuss the possible origin of the Higgs fields, belonging to the fundamental representation of the gauge group \( SU(2) \), from extra-dimensional components of the metric tensor. The opportunity of a similar origin of gauge fields is well known. The fermion sector of the SM will not be discussed.

Other variants of geometrization of the Higgs field, discussed previously, employed either additional degrees of freedom related to conformal transformations of the metric [1–3] or purely group-theoretical considerations using the method of intertwining operators [4]. Unlike that, we follow the simplest geometric approach and do not use any dynamic variables other than the multidimensional metric tensor.

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2. THE METRIC AND THE ACTION

Let there be a \( D \)-dimensional Riemannian manifold \( \mathbb{M}^D \) with the metric

\[
\begin{pmatrix}
\bar{g}_{\mu\nu}(x, y) & g_{\mu\beta}(x, y) \\
g_{\alpha\beta}(x, y) & g_{\alpha\beta}(x, z) \\
g_{\alpha\beta}(x, z) & g_{\beta\beta}(x, z) \\
\end{pmatrix}
\]

(1)

Other metric components, though maybe existing, are ignored. Here and henceforth, the indices assume the following values: \( A, B, \ldots = 1, \ldots, D \) (the full dimension is \( D = 9 + d_3 \)); \( \alpha, \beta, \ldots, \mu, \nu, \ldots = 1, \ldots, 4 \) (the quantities \( \bar{g}_{\mu\nu}(x, y) \) contain the metric of our 4D space \( M_4 \) \( g_{\mu\nu}(x) = g^{(1)}_{\mu\nu}(x) \), see Section 3); \( a, b, \ldots = 5, \ldots, 8 \) (the notation \( g_{ab} = \gamma_{ab} \) will also be used, and the space with this metric will be called \( V^4 \)); \( I, J, K = 5, \ldots, 9 \) (the subspace with the metric \( g_{IK} = g^{(2)}_{IK} \) will be denoted \( M_2 \)); \( m, n, \ldots = 10, \ldots, 9 + d_3 \) (the subspace with the metric \( g_{mn} \) will be denoted \( M_3 \)); \( i, j, k \) are used as group indices. The sets of coordinates of the subspaces \( M_4, M_2 \) (which includes \( V^4 \) and \( M_3 \) will be denoted by \( x, y, z \), respectively, and all \( D \) coordinates jointly by the letter \( Z \).

The off-diagonal components \( g_{ab}, a = 5, \ldots, 8 \), according to the standard Kaluza-Klein scheme [5], will be related to gauge fields that lie in the algebra of the symmetry group of the 4D compact factor space.
As will be shown below, the components \( g_{ab} \) contain scalar (in \( M_1 \)) fields admitting an interpretation as the Higgs field. The metric \( g_{mn} \) of the factor space \( M_3 \) is of auxiliary nature.

A basic element in this approach is the extra space \( \mathbb{V}^4 \) with the metric \( \gamma_{ab} \). It is supposed that its isometry group \( T \) causes the symmetry of the SM Lagrangian under gauge transformations. The reasons for such a high symmetry of the extra factor space are discussed in [6]; briefly speaking, they are connected with washing out the entropy from extra to usual dimensions. Our strategy will consist in choosing the group \( T \) in such a way as to provide the correct transformation law of the effective Higgs doublet that emerges from \( g_{ab} \) and belongs to the fundamental representation of the electroweak gauge group \( SU(2) \times U(1) \) of the SM.

The model dynamics will be specified by the \( D \)-dimensional action quadratic in the scalar curvature \( R_D \) of \( M_D \):

\[
S = \frac{1}{2} m_D^{D-2} \int \sqrt{g_D} D^D Z \left[ R_D + \eta R_D^2 - 2\Lambda \right],
\]

where \( \eta, \Lambda \) are constants (parameters of the theory), and \( m_D \) is the \( D \)-dimensional Planck mass.

Consider the class of linear transformations \( T \) of the coordinates \( y^a \) of the extra space \( \mathbb{V}^4 \):

\[
y'^a = T^a_b y^b, \quad a, b = 5, \ldots, 8. \tag{3}
\]

The fields \( g_{ab}(x, z) \) transform as a vector at the transformations (3). For an infinitesimal shift we accordingly have

\[
g'_{ab}(x, z) = \frac{\partial y'^a}{\partial y^b} g_{ab}(x, z) = (\delta^a_b - i\varepsilon^a_{bc}) g_{ab}(x, z). \tag{4}
\]

Let us specify the group \( T \) by requiring that (a) this group should be realized by coordinate transformations in the extra space, i.e., it should be real; (b) it should be isomorphic to the electroweak group of the SM. In other words, the group \( T \) must be built as a realification of \( SU(2) \times U(1) \). This can be done by taking as a basis the quite well-known scheme of embedding the unitary groups \( SU(n) \) into the orthogonal groups \( SO(2n) \).

We introduce the following notations for separate components of the electroweak group:

\[
\omega_1(\phi) = e^{i\phi} \in U(1); \quad \omega_2(\theta_j) = A(\theta_j) + iB(\theta_j) \in SU(2). \tag{5}
\]

Here, \( \phi \) and \( \theta_j \) \( (j = 1, \ldots, 3) \) are parameters of the groups \( U(1) \) and \( SU(2) \), respectively; \( A = \text{Re}(\omega_2), \quad B = \text{Im}(\omega_2) \) are real \( 2 \times 2 \) matrices which are the real and imaginary parts of an \( SU(2) \) element in the fundamental representation and therefore satisfy the conditions

\[
A^T A + B^T B = 1, \quad A^T B - B^T A = 0, \quad \det(A + iB) = 1. \tag{6}
\]

We also introduce the set of \( 4 \times 4 \) matrices \( T_1, T_2 \)

\[
T_1 = \begin{pmatrix} I \cos \phi & -I \sin \phi \\ I \sin \phi & I \cos \phi \end{pmatrix},
\]

\[
T_2 = \begin{pmatrix} A & -B \\ B & A \end{pmatrix}, \tag{7}
\]

where \( I \) is the unit \( 2 \times 2 \) matrix.

It is easy to verify the following statement: all possible products of the elements \( T_1 \cdot T_2 \) form a real 4-parameter group \( T \) lying in \( SO(4) \) and isomorphic to \( SU(2) \times U(1) \). As the symmetry group of the extra space \( \mathbb{V}^4 \), we choose the built group \( T = T_1 \cdot T_2 \).

It is important that the initial Lagrangian is invariant under coordinate transformations belonging to \( T \) because it is manifestly invariant under general coordinate transformations of the space \( \mathbb{V}^4 \).

It is also easily seen that the Euclidean metric

\[
\gamma_{ab} = \delta_{ab} \tag{8}
\]

is symmetric under transformations of the group \( T \). That is why it can be chosen as the metric of the basic extra space \( \mathbb{V}^4 \). The latter can be made compact, e.g., by endowing it with the topology of a torus.

3. A TRANSITION TO THE DYNAMIC VARIABLES OF THE SM

Let us connect the Higgs complex doublet \( h(x) \in \mathbb{C}^2 \) of the SM with the extra-space metric. We will try to connect the metric coefficients

\[
g_{ab} = H_a \tag{9}
\]

with the Higgs field \( h \). The latter transforms by the fundamental representation of the electroweak group \( SU(2) \times U(1) \),

\[
h' = \omega_1 \omega_2 h = (A + iB)e^{i\phi} h. \tag{10}
\]

We can express the 4-component field \( H_a \), transformed by \( T \), in terms of the two-component columns \( X, Y \):

\[
H \equiv \begin{pmatrix} X \\ Y \end{pmatrix}, \quad H' = TH = T_1 T_2 H. \tag{11}
\]

As is easily verified, the field \( \tilde{h} \) of the form

\[
\tilde{h} = X + iY, \quad X, Y \in \mathbb{R}^2, \tag{12}
\]