Plasma Gravi-Bremsstrahlung in TeV-Scale Gravity

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Abstract—We develop a theory of interaction of a classical plasma with Kaluza–Klein (KK) gravitons in the ADD model of TeV-scale gravity. The plasma is described in the kinetic approach as a system of charged particles and the Maxwell field, both confined to the brane. Interaction with multidimensional gravity living in the bulk with $n$ compact extra dimensions is introduced within a linearized theory. The KK graviton emission rates are computed taking into account plasma collective effects through two-point correlation functions of fluctuations of the plasma energy-momentum tensor. Apart from known mechanisms (such as bremsstrahlung and the gravi-Primakoff effect), we find essentially collective channels such as coalescence of plasma waves into gravitons which may be manifest in turbulent plasmas. Our results indicate that the commonly used KK gravitons production rates in stars and supernovae may be underestimated.

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1. INTRODUCTION

The TeV-scale gravity proposal due to Arkani-Hamed, Dimopoulos, and Dvali (ADD) [1] suggests that the standard model particles live in the four-dimensional subspace (the brane) of a D-dimensional bulk with $n = D - 4$ extra dimensions compactified on a torus, inhabited only by gravity. The D-dimensional Planck mass $M_{Pl}$ is supposed to be TeV-scale, and the large extra dimensions (LED) to have a sub-millimeter size. In this scenario, there is an infinite tower of massive KK gravitons [2] whose existence may be detected using table-top and collider experiments, astrophysical and cosmological observations. It has been pointed out that one of the strongest bounds on the parameters comes from the supernova SN1987A data [1–10]. Processes contributing to the energy loss due to graviton emission from SN1987A, the red giants and the Sun include photon-photon annihilation, electron-positron annihilation, nucleon-nucleon gravitational bremsstrahlung, and gravi-Compton scattering [3–10].

The previous calculations of KK graviton emission rates were performed using one-particle Feynman rules with subsequent averaging over Bose–Einstein and Maxwell–Boltzmann distributions to get results reliable for a finite temperature $T$ [4]. These calculations, however, did not take into account plasma collective effects such as Debye screening, interaction of charged particles with plasma waves etc., which may be important in astrophysical conditions. This is a goal of the present contribution. We generalize a kinetic theory of interaction of plasmas with gravitational waves, developed earlier in 3+1 dimensions [11] to the ADD model and present calculation of the gravi-bremsstrahlung rate from a non-relativistic thermal isotropic electron-ion plasma.

2. THE KINETIC APPROACH

Consider a collisionless plasma consisting of charged particles of types $\alpha$, with the parameters $e_\alpha$, $m_\alpha$ described by the microscopic distribution function [12, 13]

$$\mathcal{F}_\alpha(x, p) = \sum_{i=1}^{N_\alpha} \delta(r - r_i(t))\delta(p - p_i(t)),$$

normalized as $\int \mathcal{F}_\alpha(x, p)dp d^3x = N_\alpha$, satisfying the kinetic equation

$$\frac{\partial \mathcal{F}_\alpha}{\partial t} + v \cdot \frac{\partial \mathcal{F}_\alpha}{\partial r} + e_\alpha(E + [vB]) \frac{\partial \mathcal{F}_\alpha}{\partial p} = 0. \quad (2)$$

They interact via the electromagnetic field $E, B$ satisfying the Maxwell equations

$$\text{div}E = 4\pi \rho, \quad \text{curl}E = -\frac{\partial B}{\partial t},$$

$$\text{div}B = 0, \quad \text{curl}B = 4\pi j + \frac{\partial E}{\partial t}. \quad (3)$$
with the source terms
\[ \rho(x) = \sum_{a} e_{a} \int F_{a}(x, p) dp, \]
\[ j(x) = \sum_{a} e_{a} \int \frac{p_{i} p_{j} \delta f_{a}(x, p)}{p_{a}} dp, \quad x \equiv \{t, r\}. \] (4)

To describe gravitational radiation, we will need the 3-spatial components of the plasma energy–momentum tensor
\[ T_{ij} = mT_{ij} + F T_{ij}, \] (5)
\[ mT_{ij} = \sum_{a} \int \frac{p_{i} p_{j} \delta f_{a}(x, p)}{p_{a}} dp, \] (6)
\[ F T_{ij} = -\frac{1}{4\pi} \left( E_{i} E_{j} + B_{i} B_{j} - \delta_{ij} \frac{E^{2} + B^{2}}{2} \right), \] (7)
where \( p_{a} = \sqrt{p^{2} + m_{a}^{2}} \).

To solve the set of equations (2)–(4), we use perturbation theory in terms of the electric charges. First we separate fluctuations from the mean distribution using the approach of [11, 12]:
\[ F(t, r, p) = f_{0}^{a} + \delta f_{a}^{0} + \delta f_{a}, \] (8)
where \( f_{0}^{a} \equiv \langle F_{a}(x, p) \rangle \) is the equilibrium Maxwell distribution function:
\[ f_{0}^{a} = \frac{N_{a}}{(2\pi v_{Ta}^{2})^{3/2}} e^{-(v^{2}/v_{Ta}^{2})/2}, \]
\[ v_{Ta} = \frac{T_{a}}{m_{a}}, \quad v_{Ta} \ll 1, \] (9)
\( \delta f_{a}^{0} \) represents fluctuations due to chaotic particle motion (“zero” fluctuations), while \( \delta f_{a}(t, r, p) \) stands for fluctuations due to electromagnetic interaction of the particles. Using the fact that \( f_{0}^{a} \) satisfies the free equation (for zero charges), we obtain:
\[ \frac{\partial \delta f_{a}}{\partial t} + v \frac{\partial \delta f_{a}}{\partial r} + e_{a} (E + \nu B) \frac{\partial (f_{0}^{a} + \delta f_{a}^{0} + \delta f_{a})}{\partial p} = 0. \] (10)
We then further expand the fluctuation \( \delta f_{a} \) in power series in terms of charges: \( \delta f_{a} = \delta f_{a}^{1} + \delta f_{a}^{2} + ... \), and use the Fourier transformation
\[ f(x) = \frac{1}{(2\pi)^{D}} \int f(k) e^{-ik_{\mu}x^{\mu}} d^{D}k \]
to get in the first two orders
\[ \delta f_{a}^{1}(k, p) = -\frac{i e_{a}}{(\omega - k \nu)} F(k) \frac{\partial f_{0}^{a}}{\partial p}, \]
\[ F \equiv E + [\nu B], \] (11)
\[ \delta f_{a}^{2}(k, p) = -\frac{i e_{a}}{(2\pi)^{4}(\omega - k \nu)} \int F(k - k_{1}) \]
\[ \times \frac{\partial}{\partial p} (\delta f_{a}^{0}(k_{1}, p) + \delta f_{a}^{1}(k_{1}, p)) d^{4}k_{1}. \] (12)

The corresponding expansion of the charged particles’ energy–momentum tensor will read
\[ mT_{ik} = mT_{0}^{0} + \delta mT_{ik}^{0} + \sum_{l=1}^{\infty} \sum_{a} \int \frac{p_{i} p_{l} \delta f_{a}^{l}(x, p)}{p_{a}} dp. \] (13)

The first nonzero contribution to the gravitational radiation comes from terms of the second order in the fields \( E, B \), namely, \( \delta T_{ik} = \delta mT_{ik}^{2} + \delta F T_{ik} \):
\[ \delta T_{ik} = -\frac{1}{(2\pi)^{4}} \int d^{4}k_{1} D_{ik}\delta \sum_{\alpha} F_{\alpha}(k_{1}, p) \]
\[ + \frac{1}{4\pi} \left( \delta_{l} \delta_{k} \epsilon_{k_{0}l_{0}k_{2}} \omega_{m} \right) E_{r}(k_{1}) \]
\[ \Delta_{iks} \delta F_{\alpha}(k_{1}, p) \]
\[ = i e_{a} \int d^{3}p_{k} p_{l} \omega_{k} \chi_{rs}(k_{2}) \frac{\delta F_{\alpha}(k_{1}, p)}{\partial p_{r}}, \quad \chi_{rs} = \frac{1}{k_{2}} (k_{2} v_{s} - (k_{2} v) \delta_{rs}). \] (17)

3. Gravitational Radiation

In the ADD model one considers [2] the linearized \( D = 4 + n \)-dimensional Einstein equations
\[ \Box \psi_{MN} = \frac{x_{0}^{2}}{2} T_{MN}, \]
where \( \psi_{MN} = h_{MN} - \eta_{MN} h_{0}^{0}/2, \quad h_{MN} = g_{MN} - \eta_{MN}, \) the indices \( M, N \) run over the brane, \( \mu, \nu = 0, 1, 2, 3 \) and \( n \) over the directions on the torus, and the harmonic gauge \( \delta_{N} \psi_{MN} = 0 \) is understood. A formula for the total (integrated over space and time) energy loss to gravitational radiation in the ADD model was recently derived in [14]:
\[ \mathcal{E} = \frac{x_{0}^{2}}{16\pi^{3} V_{d}} \sum_{N \in \mathbb{Z}^{d}} \int d^{4}k \left. T_{SN}(k) T_{LR}(k) \tilde{\lambda}^{SNLR} \right|_{k_{0}}^{0}, \]
\[ k_{0} = \sqrt{|k|^{2} + (2\pi N/L)^{2}}, \]
\[ V_{d} = (2\pi L)^{D-4}, \] (18)
where \( T_{SN}(k) \) is the four-dimensional Fourier transform, and
\[ \tilde{\lambda}^{SNLR} = \frac{1}{2} \left[ \eta^{SL} \eta^{NR} + \eta^{SR} \eta^{NL} \right] \]