Numerical investigation of the perfect gas behavior in a vibrating cylindrical cavity with thermally insulated walls*

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The influence of the vibrational action on a cylindrical cavity filled with a viscous perfect gas is studied numerically. The problem is solved in a two-dimensional axisymmetric statement. The cavity walls are thermally insulated. The numerical solution of the problem is compared with the analytic one in the linear approximation. The nonlinear effects and the non-one-dimensional effects are described.

Key words: vibration amplitude, vibration frequency, perfect gas, shock wave, regime of stationary oscillations.

Introduction

Vibration is a fairly frequently encountered phenomenon in technology, which may be both an accompanying factor and part of technological process. A detailed study of the vibrational effect on closed cavities with gas is of practical importance. A number of experimental, theoretical, and numerical studies are devoted to various vibrational effects on the cavities (tubes) [1–12]. The longitudinal oscillations of gas in closed tubes with a vibrating piston have been widely studied. The wave processes in the Kundt’s tube were studied experimentally and theoretically (in the one-dimensional statement) in the work [1] at a small amplitude of piston vibrations and a frequency close to the natural frequency of the system. In the work [2], the wave processes were described in the one-dimensional statement using the numerical integration of gas dynamics equations, the processes of a transition from the initial state to a periodically repeating regime without rigid constraints for the vibration amplitude were considered. The vibration frequency was taken in the neighborhood of the halved natural frequency of the system. Nonlinear resonances of the second and third orders were investigated experimentally in the work [3], a transition from nearly harmonic gas oscillations to strongly nonlinear ones was considered. The acoustic flow in a rectangular cavity with a piston oscillating at a constant frequency was studied numerically in the work [4]. The cavity was filled with

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air, the cavity walls were adiabatic. The amplitude of piston oscillations was varied. It was shown that the acoustic flow structure changes significantly with increasing amplitude of oscillations. The acoustic flow in a rectangular cavity with differently heated horizontal boundaries and adiabatic vertical boundaries was studied numerically in the work [5]. The wave motion was caused by a vibration of the cavity left boundary with a frequency leading to the formation of standing acoustic waves. The vibration influence on thermal convection was described. Thermal convection at the expense of a joint action of the gravity force and vertical vibration in a square cavity with differently heated vertical walls and adiabatic horizontal walls was investigated numerically in the works [6, 7], and the entire cavity was subjected to vibration. The influence of the vibration frequency and the Rayleigh number on thermal convection was established. A square cavity with adiabatic horizontal boundaries and differently heated vertical boundaries was considered in the work [8]. The cavity was in the gravity force field, the volumetric force varying according to a sine law was specified, which acted in the horizontal or vertical direction. The acoustic flow formed by standing waves in a tube of arbitrary width was studied analytically in the work [9] with regard for thermal conductivity and temperature dependence of viscosity. The problem solution was obtained in the linear approximation. Both the planar two-dimensional case and axisymmetric case were considered.

A numerical investigation of wave processes in a cavity subjected to the vibrational effect was done in the work [10] in the one-dimensional statement. The maximum temperature was determined, which was reached inside the cavity at a vibration with a frequency from a chosen range, a complex wave motion at the process initial stage was described. The regime of stationary oscillations for two types of the boundary conditions: the adiabatic and isothermal ones was described at the same problem statement in the work [11], and the nonlinear effects under the adiabatic boundary conditions were determined in [12].

In the present work, the influence of the vibrational effect on a cylindrical cavity with adiabatic boundaries, which is filled with a viscous perfect gas, is studied in a two-dimensional statement. The range of vibration frequency includes both weak effects, which may be described with the aid of a linear theory, and strong effects giving rise to nonlinear effects. Computational results are compared with the analytic solution of the problem in the linear approximation of [9, 13] as well as with the results of the computation of a problem in the one-dimensional statement [12] (in the absence of the lateral surface of the cylindrical cavity).

Problem statement

Consider a cylindrical cavity (tube) of length \( L \) and diameter \( 2M \) with impermeable butt ends (Fig. 1). Let \( (x', r') \) be a fixed reference frame, \( (x, r) \) is a reference frame co-moving with the vibrating cavity. The cavity is filled with a perfect viscous gas (air). The gas in the cavity is initially in the state at rest and at a constant temperature \( T_0 \) and constant pressure \( p_0 \). The system is deviated from the equilibrium by a vibration effect \( A \cos(\omega t) \) with the constant amplitude \( A \) and frequency \( \omega \). The adiabatic conditions are specified on the surface of the cylinder and on its butt ends. The coefficients of thermal conductivity, heat capacity, and viscosity are assumed constant.

The system of equations governing the gas motion with respect to the vibrating cavity has the following form in a cylindrical coordinate system:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{1}{r} \frac{\partial \rho r v}{\partial r} = 0,
\]

\[
\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) \right) + \frac{\mu}{3} \left( \frac{\partial^2 u}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) \right) +
\]

\[
+ \rho A \omega^2 \cos(\omega t),
\]