INTRODUCTION

The Navier–Stokes equations for a viscous compressible heat-conducting gas are a base model for solving wide classes of problems in aero- and fluid dynamics. Under various assumptions on the character of the flows in question, it is easy to obtain from them various simplified models, such as the Euler equations, boundary layer equations, parabolized equations, etc. For this reason, the construction of effective and economic numerical algorithms for the Navier–Stokes equations is still an important task, despite the variety of numerical methods (see, e.g., [1–6]). As a rule, solutions to these equations are characterized by steep-gradient areas and other singularities, such as boundary layers and detached shocks, separation zones, etc. Therefore, severe requirements have to be imposed on the numerical algorithms used. These algorithms have to be sufficiently accurate and stable, conservative, and efficient, so that solution can be obtained in a reasonable time on available computers. Explicit difference schemes as applied to the Navier–Stokes equations is ineffective because of the severe restrictions imposed on mesh size, especially when a stationary solution is determined by the relaxation method. For this reason, most frequently used are implicit difference schemes, which are free from the stability restrictions or involve weaker constraints. A review of the most common difference schemes can be found, for example, in [1–6].

Implicit difference schemes are usually based on factorization and splitting methods (see [2–7]), which reduce the original multidimensional problems to the sequential (or parallel) solution of their one-dimensional analogues. When one-dimensional problems are solved using implicit schemes, we have to invert matrices, whose dimension increases with the number of equations and the problem dimension. As a result, this leads to a significant increase in the number of arithmetic operations per grid node. Due to splitting used in one-dimensional problems, this shortcoming can be overcome and the implementation of schemes at fractional steps is reduced to scalar tridiagonal Gaussian elimination or to running schemes (see, for example, [2, 7]). However, the splitting (or factorization) of the operators in the original multidimensional problem leads to additional terms (namely, dissipative and higher order ones) appearing in the difference scheme. As a result, the properties of the numerical algorithm degrade. Therefore, when economic algorithms are designed, the operators have to be split so as to minimize the effect of these terms.

We propose difference schemes based on the splitting of the original multidimensional Navier–Stokes equations into their one-dimensional analogues with the subsequent splitting of the one-dimensional problems so that the implementation of the schemes at fractional steps is reduced to scalar tridiagonal Gaussian elimination; i.e., they are efficient in terms of the number of operations per grid node. The difference schemes constructed are unconditionally stable or have weak stability constraints, while the effect of splitting is minimal; i.e., their properties are close to those of unfactorized schemes. A minimum dissipation scheme for one-dimensional gasdynamic simulations was proposed in [8]. We propose difference schemes of approximate factorization and predictor–corrector schemes for the numerical solution of the Navier–
Stokes equations in Cartesian and curvilinear coordinates. The algorithms are validated in test computations. Numerical results are presented for viscous gas flows in a plane channel with blowing on its surface and for the flow past a complex-geometry body that simulates the flow in an air vent.

1. DIFFERENCE SCHEMES FOR ONE-DIMENSIONAL EQUATIONS

Difference schemes based on the splitting of operators are first constructed for the one-dimensional gasdynamic and Navier–Stokes equations in Cartesian coordinates and, then, they are generalized to the multidimensional case. The Navier–Stokes equations are represented in the conservative form

\[
\frac{\partial U}{\partial t} = -\frac{\partial W}{\partial x},
\]

where

\[
U = \begin{pmatrix} \rho \\ \rho v \\ E \end{pmatrix}, \quad W = \begin{pmatrix} \rho v \\ \rho v^2 + p - \mu \frac{\partial v}{\partial x} \\ \nu \rho H - \kappa \frac{\partial T}{\partial x} - \mu \frac{\partial v}{\partial x} \end{pmatrix}, \quad E = \rho \left( e + \frac{v^2}{2} \right), \quad H = E + p.
\]

To make system (1) closed, we introduce the equation of state

\[
p = p(\rho, e), \quad e = e(T) = c_v T
\]

and specify the viscosity and heat conductivity as functions of temperature: \( \mu = \mu(T) \) and \( \kappa = \kappa(T) \). When \( \mu = \kappa = 0 \), system (1) turns into the gasdynamic equations. Along with the conservative form, the original equations (1) are represented in the nonconservative and limit-conservative forms

\[
\frac{\partial f}{\partial t} = -B f + F \quad \text{or} \quad \frac{\partial f}{\partial t} = -A^{-1} W, \quad A = \frac{\partial U}{\partial f}.
\]

Note that the Navier–Stokes equations written in conservative form (1) are equivalent to those written in nonconservative form (3). Obviously, the choice of \( f \) specifies the form of the matrix operators \( B \). As a result, we can consider various classes of difference schemes. For gasdynamic and Navier–Stokes equations, the unknown functions are usually the density, velocity, and pressure or temperature, which is determined by the boundary conditions specified. However, the form of \( B \) differs considerably for different components of \( f \). Indeed, for the equation of state \( p = (\gamma - 1) e \rho \), the matrix \( B \) is

\[
f = \begin{pmatrix} \rho \\ v \\ T \end{pmatrix}, \quad B = \begin{pmatrix} \frac{\nu}{\rho} \frac{\partial}{\partial x} & \frac{\rho}{\rho} \frac{\partial}{\partial x} & 0 \\ a^2 \frac{\partial}{\partial x} & \frac{\nu}{\rho} \frac{\partial}{\partial x} - \frac{1}{\rho} \frac{\partial}{\partial x} & b^2 \frac{\partial}{\partial x} \\ 0 & c^2 \frac{\partial}{\partial x} & \frac{\nu}{\rho} \frac{\partial}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial x} \end{pmatrix}, \quad F = \begin{pmatrix} 0 \\ 0 \\ \frac{\mu}{\rho} \left( \frac{\partial v}{\partial x} \right)^2 \end{pmatrix},
\]

where

\[
a^2 = \frac{1}{\rho} \frac{\partial p}{\partial \rho}, \quad b^2 = \frac{1}{\rho} \frac{\partial p}{\partial T}, \quad c^2 = \frac{p}{\rho c_v},
\]