The harmonic expansion method is one of the mathematical methods to forecast meteorological data series used, e.g., in papers [1, 2]. The method’s main idea is to choose for the analyzed series a sum of elementary harmonics which overlap one another with different phase shifts and amplitudes. To find out the numerical values of harmonics data amplitudes and phase shifts, we used the least-squares method. The periods of the harmonics were found from Fourier transformation of the analyzed series.

In papers [1, 2], this method was tested on a meteorological data series for the Yorkuta region. For further analysis, one should select from the given geographical region the most extensive surface air temperature (SAT) series. We have selected the surface air temperature series of Syktyvkar since its length is 113 years (from 1895 to 2007).

Directly in paper [1], the harmonic expansion method was applied to a series with a 10-year smoothing period. One of the reasons why this smoothing period was chosen by the authors of paper [1] is the decreased impact of local anomalous values on cyclic specific features of average annual SAT behavior. This smoothing interval was also applied to the average annual temperature series for Syktyvkar.

The analyzed 10-year moving-mean series for average annual temperature in Syktyvkar, the linear trend was set aside and excluded

\[ y(x) = 0.6 + 0.0057(x - 1949), \quad (1) \]

where \( x \) is time measured in years.

This trend fits the development scenario for the north Arctic region until 2080 presented in the Third Report by the Intergovernmental Panel on Climate Change [3].

Figure 1 shows the average annual SAT series for Syktyvkar (firm line), the 10-year smoothing of this series (pecked line), and the 10-year smoothing excluding the linear trend (dotted line).

Then, for comparison, we applied to the smoothed series without a trend the harmonic expansion and wavelet transformation procedures.

Wavelet transformation of a signal [4] is essentially its expansion on the basis constructed from a special generating function (wavelet) by means of large-scale changes and transfers. Thus, a one-dimension signal is projected on the time-frequency plane in the form of a two-dimensional distribution of continuous wavelet transformation coefficients.

\[ W(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \psi_{ab}^*(t) dt, \quad (2) \]

where \( W(a, b) \) are wavelet transformation coefficients; \( f(t) \) is an explored function; \( t \) is time; \( \psi_{ab} \) is a wavelet; \( a \) is a scale; \( b \) is the parameter of the shift along the time axis; and \( ^* \) is the conjugation operator.

The inverse to (2) transformation has the form
where \( C_\psi \) is a normalizing coefficient.

First, we shall conduct an analysis and make a forecast for this series by the harmonic expansion method. Based on the analysis of the Fourier series’ coefficients (Fig. 2), we can set aside the periodicities of 11 and 14 years, and 22 and 37 years.

The resulting harmonic expansion of this series has the following form:

\[
T(t) = -0.003 + 0.02\sin\frac{2\pi(t + 2.27)}{11} + 0.15\sin\frac{2\pi(t + 4.81)}{14} + 0.12\sin\frac{2\pi(t + 6.71)}{22} - 0.11\sin\frac{2\pi(t - 2.91)}{37},
\]

where \( t \) is time measured in years from 1895.

Figure 3 gives a comparison of the harmonic expansion function (4) with the analyzed series and shows a forecast for the series by means of expanding the selected harmonic function (4) until 2060.

The result of the wavelet transformation for the 10-year moving-mean series without a trend is shown in Fig. 4 (the Morlet wavelet was used in [5, 6]).

Analysis of the wavelet coefficients map in Fig. 4 yields the same periodicities as the Fourier transformation (see Fig. 2); however, several remarks can be made pertaining to the time periods when fluctuations of a different scale occur:

1. The harmonic with the 11-year period occurs only in the time interval of 1940–1965, while the harmonic with the 14-year period occurs, on the contrary, only outside the time interval of 1940–1965.

2. The harmonic with a 22-year period occurs only since 1955.