Fluctuations are intrinsic to the majority of physical processes. They reflect the microscopic nature of interactions of particles forming the corresponding system. Under equilibrium conditions, fluctuation amplitudes obey the Gaussian distribution and exhibit the Lorentz power spectrum. For low frequencies, the power spectrum is constant, whereas for high frequencies, it drops in inverse proportion to the frequency squared. In a number of systems, fluctuations are observed whose power spectrum is inversely proportional to the frequency within the range extended for several orders of magnitude: $S(f) \sim \frac{1}{f}$ [1]. These systems are usually considered to be rather complicated, with their certain parameters being far from the state of equilibrium. Steady random processes are characterized by critical dynamics and by scale-invariant fluctuation distributions. In these systems, the fluctuation energy can be accumulated so that low-frequency catastrophic spikes are possible.

The formal mathematical description of $\frac{1}{f}$ fluctuations is presented in the form of the fractional integration of white noise [2]. However, this description is hardly connected to the physical features of the system. Fluctuations of thermodynamic quantities near a critical point manifest scale-invariant properties [3]. This point is determined by the conditions of the equalizing features of different phases. At the critical point, scale-invariant fluctuations are developed without approaching any fixed state of equilibrium. For turbulent fluid flows, the dynamic development of fluctuations within a wide-scale range is of characteristic as well [4]. The mechanisms of the appearance of $\frac{1}{f}$ fluctuations can be characterized by the concept of a self-organized criticality [5], which describes the avalanche dynamics. On the basis of cellular automates, the critical behavior of various systems was demonstrated for a large number of computer models. They included a sand heap [5], biological evolution [6], and interphase dynamics [7]. Systems exhibiting properties of self-organized criticality show a stable spatial and stable time scale-invariant state without finely tuning their parameters.

In [8], random processes that obey the $\frac{1}{f}$ fluctuation spectrum and the scale-invariant distribution were simulated by a set of nonlinear stochastic equations describing nonequilibrium phase transitions:

$$\frac{d\phi}{dt} = -\phi \psi^2 + \psi + \Gamma_1(t),$$
$$\frac{d\psi}{dt} = -\psi \phi^2 + 2\phi + \Gamma_2(t),$$

(1)

where $\phi$ and $\psi$ are dynamic variables and $\Gamma_1$ and $\Gamma_2$ are $\delta$-correlated noises that obey the normal distribution.

The criticality is determined by the white-noise level that corresponds to the noise-induced transition [9]. In the course of simulating processes by stochastic equations, the principle of the entropy maximum can be chosen as a stability criterion. The solutions to the stochastic equations are probabilistic functions that make it possible to find Shannon’s information entropy [10]:

$$H = -\sum_n P_n \ln P_n.$$  

(2)

The distribution functions $P_n$ should be normalized: $\sum_n P_n = 1$. Here, subscript $n$ relates to the sequence of the division of a distribution-function argument over its magnitude. In essence, the information entropy has the properties of the statistical Gibbs entropy [11]. However, it is also used in the statistics of biological, communicative, and social systems. At the same time, for complicated physical systems characterized by a
Entropy maxima and values of parameter \( q \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Gibbs–Shannon entropy ( H )</th>
<th>Tsallis entropy ( H^T )</th>
<th>Renyi entropy ( H^R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>( P(\psi^2) )</td>
<td>( P(\varphi^2) )</td>
<td>( P(\varphi^2) )</td>
</tr>
<tr>
<td>( H_{\max} )</td>
<td>2.242</td>
<td>19.301</td>
<td>18.412</td>
</tr>
<tr>
<td>( q )</td>
<td>0.62</td>
<td>0.62</td>
<td>0.62</td>
</tr>
</tbody>
</table>

scale-invariant distribution with power-law tails, we may consider that the Gibbs statistics does not provide consistency with the principle of the entropy maximum [11, 12].

A method for solving this problem was given by Tsallis [13], who proposed to use a deformed logarithmic function. The Tsallis entropy is determined by the expression

\[
H^T = \frac{1}{1-q} \left( \sum_n p_n^q - 1 \right) \tag{3}
\]

For power-law distribution functions, employment of the Renyi entropy is also proposed in [14]:

\[
H^R = \frac{1}{1-q} \ln \sum_n p_n^q \tag{4}
\]

Both entropies \( H^T \) and \( H^R \) are decent on the parameter \( q \) whose value is determined by the physical properties of the system.

In the author’s opinion, a complicated system can be characterized by a unique distribution. A sufficiently detailed theory of a complicated system should contain a set of nonlinear stochastic equations corresponding to the hierarchy of subordination and control. The solutions to this set of equations are distribution functions that can have both power-law (non-Gaussian) and Gaussian tails.

The problem of the existence of the entropy maximum should be solved with allowance for the analysis of the control and subordination hierarchy of equations describing the system [15]. The approach of Tsallis [13] or Renyi [14] can be applied in the case of limited information on the system under study. For example, in experimental investigations, we often have to deal with a unique distribution, whereas distributions for other quantities of the system can turn out beyond the limits of accuracy of the available instruments. In this case, for calculations, it is important to have estimates of parameter \( q \) that enter into the expression for Tsallis entropy (3) and Renyi entropy (4).

Various distribution functions of dynamic quantities also exist for the set of stochastic equations (1). The first equation in (1) has a distribution function with a power-law tail [15]. The second equation has a distribution function that drops with increasing the argument in the same manner as is the case for the Gaussian distribution. The analysis of the control and subordination performed in [15] has shown that the second equation in set (1) is the controlling one. Therefore, the distribution function of the second equation can be employed for finding the maximum of the Gibbs–Shannon information entropy (2). The entropy for the distribution function \( P(\psi^2) \) was calculated for different values of the white-noise intensity \( \sigma \). It was found that the position of the entropy maximum determines the critical state of the system, provided the white-noise intensity in equations (1) corresponds to a noise-induced transition. In this case, spectra of fluctuating quantities are inversely proportional to the frequency. The existence of the entropy maximum not only explains tuning for the criticality but testifies to the stability of modes possessing the \( \frac{1}{f} \) fluctuation spectrum. In this case, insofar as the characteristic features of critical fluctuations are distribution functions with power-law tails, i.e., \( P(\varphi^2) \) and \( P\left(\frac{1}{\psi^2}\right) \), expressions (3) for Tsallis entropy and expressions (4) for Renyi entropy were used. The condition of the coincidence of coordinates for the maxima of Gibbs–Shannon entropy, Tsallis entropy, and Renyi entropy yields the values of the parameter \( q \) that enters into the expression for the definition of these quantities. The results obtained are listed in the table.

The fundamental principle of the entropy maximum can be applied to estimate the stability of the resulting process arising in the course of the interaction of a random process obeying the \( \frac{1}{f} \) spectrum and of another random process or in the case of determinate action on it. Insofar as it is impossible to fix all parameters, in the course of interaction, random processes can exchange their energy with each other. Thus, the resulting process that corresponds to the entropy maximum will become stable. The same can be said for the determinate action on random processes. In this case, energy exchange also is possible so that the entropy maximum corresponds to a stable process. For example, sometimes, it is proposed to use the action of a harmonic force to decrease the effect of dangerous low-frequency spikes in a system generating random pulsations that obey the \( \frac{1}{f} \) spectrum. We now consider this proposition in more detail.